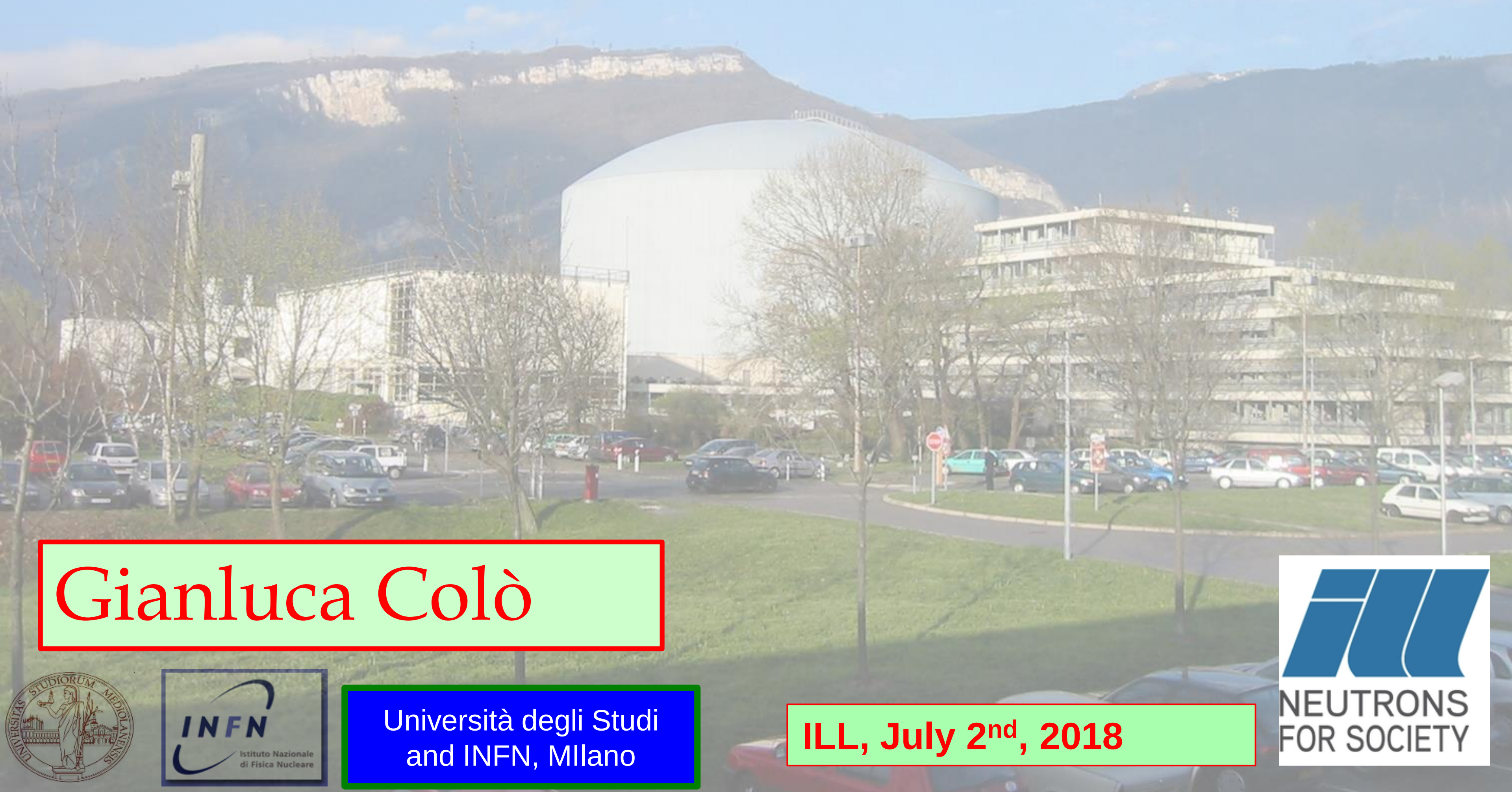
Atomic nuclei as building blocks of the interdisciplinary quantum many-body science

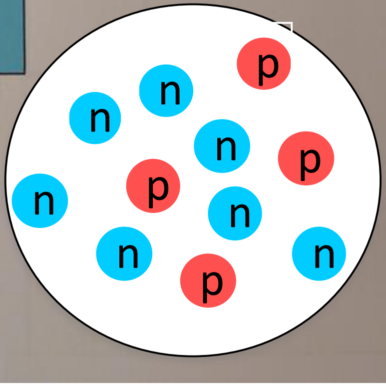


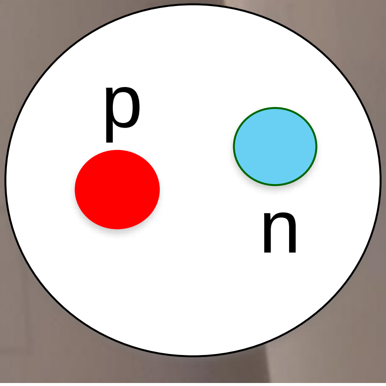
Gianluca Colò

Università degli Studi
and INFN, MIlano

ILL, July 2nd, 2018

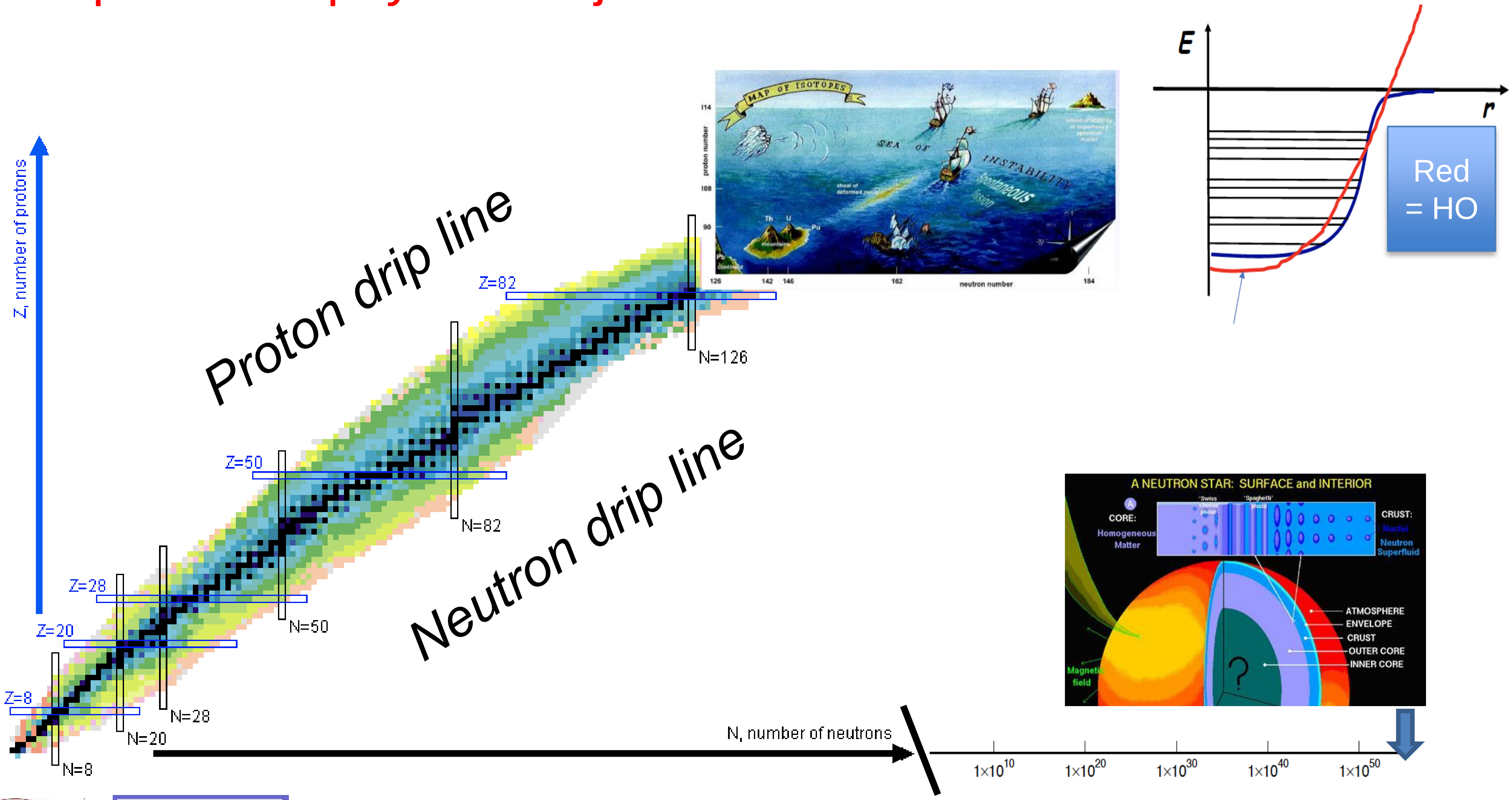
- **Which nuclei, i.e. which aggregates of neutrons and protons, exist?**
- **How do the complex phenomena of nuclear physics emerge?** *(Nuclei are made up by nucleons but often behave as a whole, they are stiff under compression but look different when they undergo fission...)*
- **Which is the role of the mutual interaction and which is the role of many-body correlations?** *(Common to other quantum many-body systems).*
- **How were atomic nuclei made?** *(The nucleosynthesis problem)*





Discovering the limits of the nuclear existence is still underway

Three **new territories**: nuclei at the drip lines, super-heavies, compact astrophysical objects



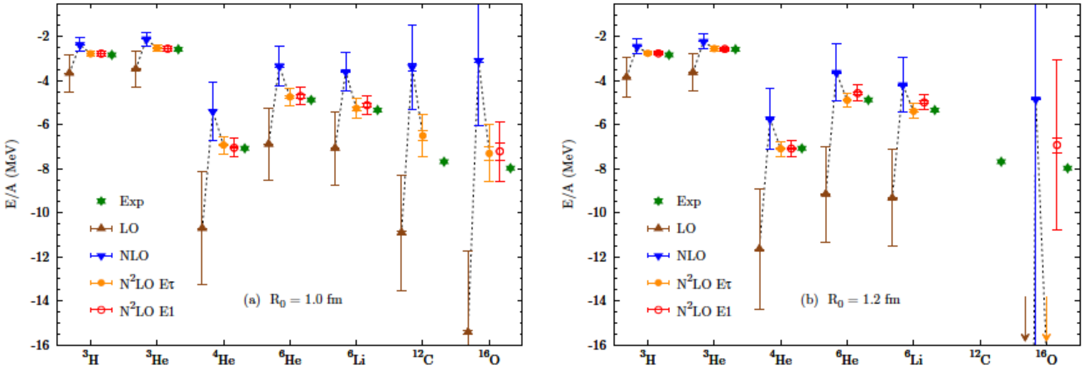
Theory: *ab initio* ?

- There are attempts to derive the interaction between nucleons from the dynamics of quarks inside nucleons (**Quantum Chromo Dynamics, QCD**).

No bound two- or three-nucleon system.
BE (^{4}He) around 5 MeV.
T. Inoue *et al.*, PRL 111, 112503 (2013)

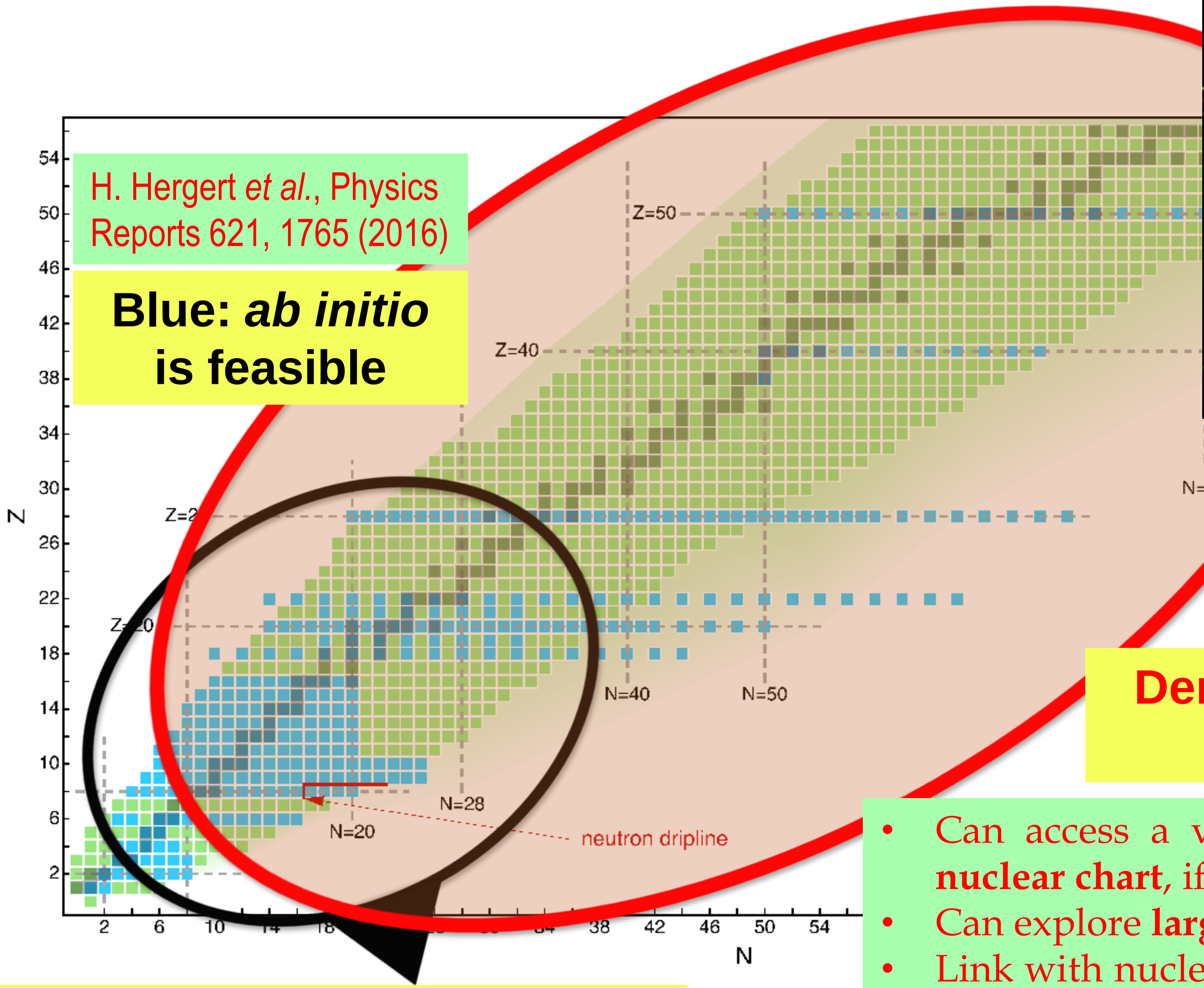
- Most often, **effective** theories are used. (Very) demanding from the computational viewpoint.

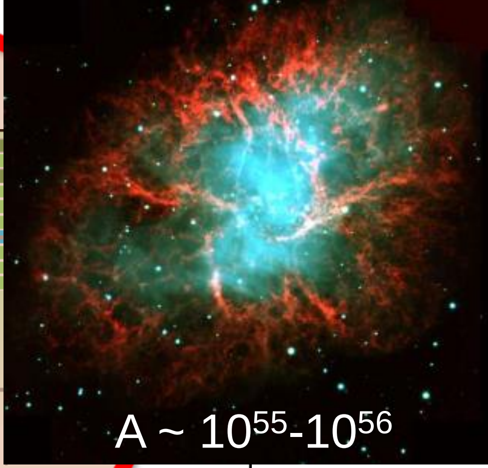
D. Lonardoni *et al.*, PRC 97, 044318 (2018)



H. Hergert *et al.*, Physics Reports 621, 1765 (2016)

Blue: *ab initio* is feasible



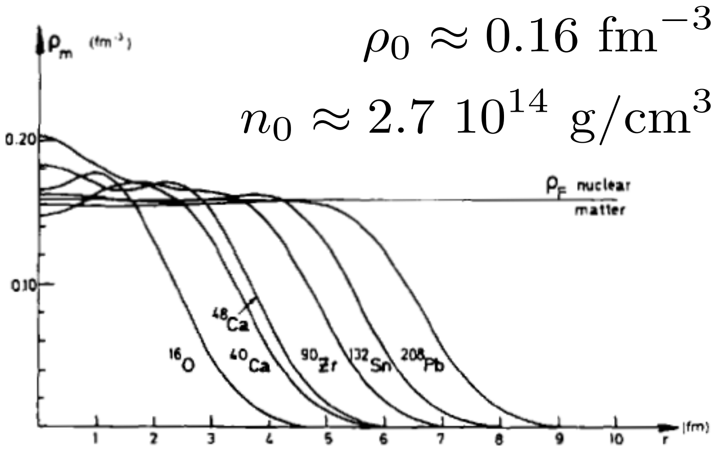

$A \sim 10^{55}\text{-}10^{56}$

Density Functional Theory, DFT

- Can access a very large part of the **nuclear chart**, if not all
- Can explore **large E_x**
- Link with nuclear matter and compact objects like **neutron stars**

Shell model (Configuration interaction, CI)

Why does DFT work if the force is **strong**?

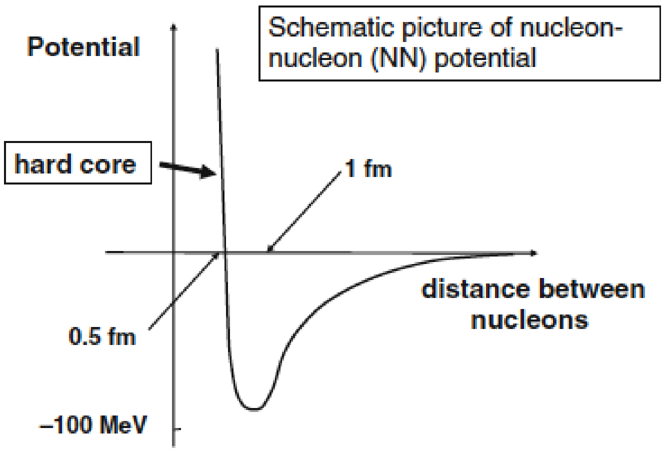


$$\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$$

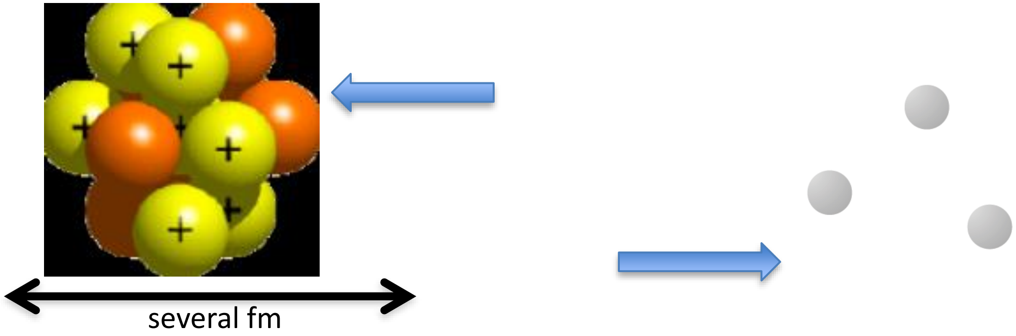
$$n_0 \approx 2.7\ 10^{14}\ \mathrm{g/cm}^3$$

$$\boxed{\frac{1}{\rho} = \frac{4}{3}\pi r_0^3}$$

- The nuclear system is rather **dilute**: distance r_0 between particles > interaction range.
- Nucleons do not feel so often the strongest part of V_{NN}.



Schematic picture of nucleon-nucleon (NN) potential



Nuclear Density Functional Theory

$$E = \int \mathcal{E}[\rho] \; d^3r$$

Energy density = $\mathcal{E}$
E is a functional

"DFT is an exactification of Hartree-Fock" (W. Kohn).

Courtesy: M. Pössel

- Mechanism for saturation
- Flexibility (i.e., enough terms) to account for the many features of finite nuclei
- Suitable for extensive computational efforts
- "Guaranteed" by the Hohenberg-Kohn theorem (1964)

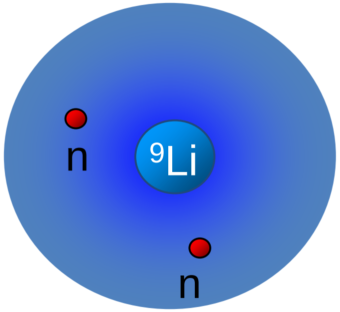
$$E_{\text{ground state}} = \min_\rho E[\rho]$$

The **exact** functional has a minimum at the exact ground-state density, where it assumes the exact energy as a value.

The principle is relatively **simple**: write the energy of a system as a functional of the density and minimize !

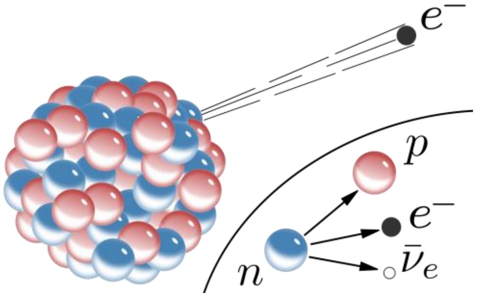
- Masses $$M(N,Z) = Zm_p c^2 + Nm_n c^2 - BE(N,Z)$$
- Radii

Halo nucleus



From: T. Nakamura

- Deformations
- Excited states: vibrations, rotations
- Decays (for instance, β and ββ transitions)



…

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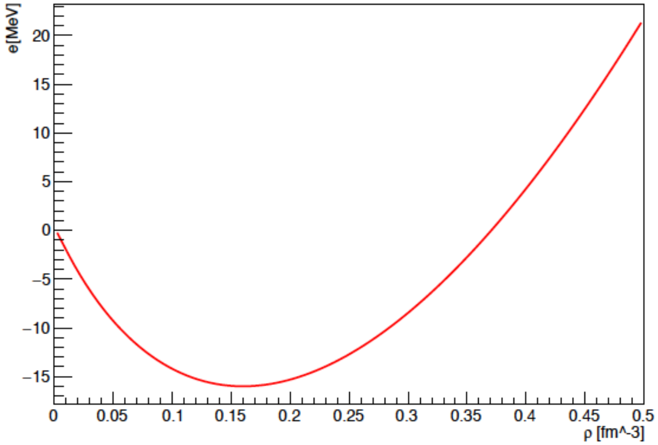
ILL Grenoble, July 2nd, 2018

Simplistic / realistic functional(s)

$$
\begin{aligned}
\mathcal{H} &= \frac{\hbar^2}{2m}\tau \\
&+ \frac{1}{4}t_0[(2+x_0)\rho^2 - (2x_0+1)(\rho_p^2+\rho_n^2)] \\
&+ \frac{1}{24}t_3\varrho^\sigma[(2+x_3)\varrho^2 - (2x_3+1)(\rho_p^2+\rho_n^2))] \\
&+ \frac{1}{8}[t_1(2+x_1)+t_2(2+x_2)]\tau\rho \\
&+ \frac{1}{8}[t_2(2x_2+1)-t_1(2x_1+1)](\tau_p\rho_p+\tau_n\rho_n) \\
&+ \frac{1}{32}[3t_1(2+x_1)-t_2(2+x_2)](\vec{\nabla}\rho)^2 \\
&- \frac{1}{32}[3t_1(2x_1+1)+t_2(2x_2+1)][(\vec{\nabla}\rho_p)^2+(\vec{\nabla}\rho_n)^2] \\
&+ \frac{1}{2}W_0[\vec{J}\cdot\vec{\nabla}\rho+\vec{J}_p\cdot\vec{\nabla}\rho_p+\vec{J}_n\cdot\vec{\nabla}\rho_n] \\
&- \frac{1}{16}(t_1x_1+t_2x_2)J^2+\frac{1}{16}(t_1-t_2)(J_p^2+J_n^2).
\end{aligned}
$$



$$
\tau = \sum_i |\vec{\nabla}\phi_i|^2 \qquad \vec{J} = \sum_{i,\sigma,\sigma'} \phi_i^\dagger \vec{\nabla}\phi_i \times \langle\sigma'|\vec{\sigma}|\sigma\rangle
$$

- **Skyrme: local functionals**
- **Gogny: non local f.**
- **Relativistic EDFs: covariant formulation**

Chemical Accuracy

$$\rho = \sum_i |\phi|^2 \qquad \tau = \sum_i |\vec{\nabla}\phi|^2$$

exact exchange and exact partial correlation

exact exchange and compatible correlation

meta-generalized gradient approximation

$$E_{\text{xc}}^{\text{mGGA}} = \int d^3r \; f(\rho, \nabla\rho, \nabla^2\rho, \tau)$$

generalized gradient approximation

$$E_{\text{xc}}^{\text{GGA}} = \int d^3r \; f(\rho, \nabla\rho)$$

local spin density approximation

$$E_{\text{xc}}^{\text{LDA or LSDA}} = \int d^3r \; \rho(r) e_{\text{xc}}^{\text{unif}}[\rho]$$

World

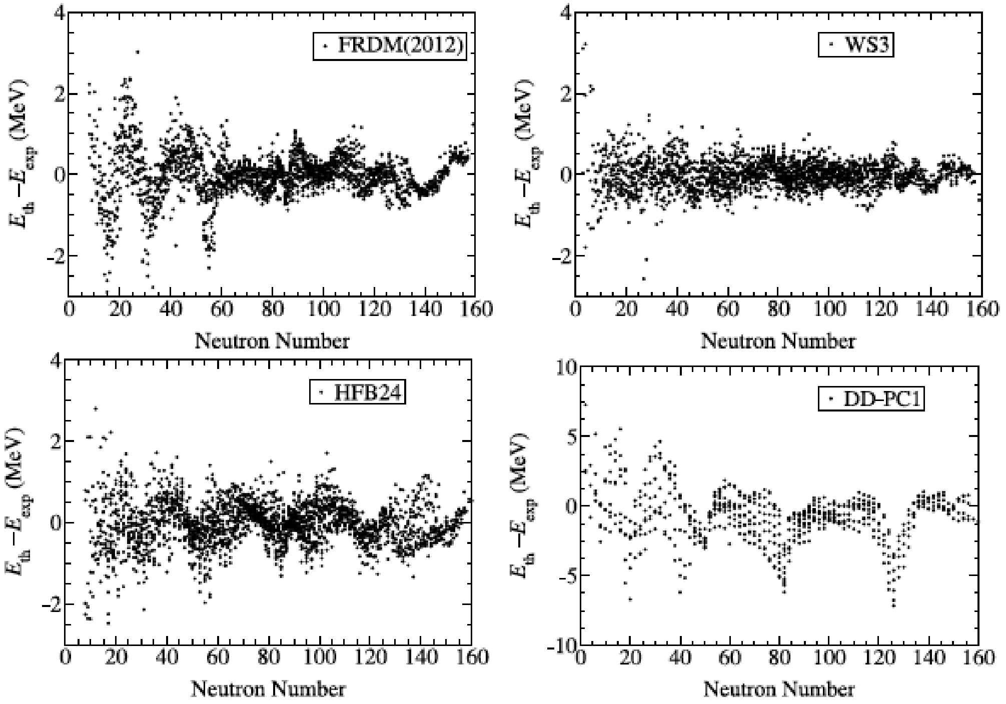
FIGURE 1. Jacob's ladder of density functional approximations. Any resemblance to the Tower of Babel is purely coincidental. Also shown are angels in the spherical approximation, ascending and descending. Users are free to choose the rungs appropriate to their accuracy requirements and computational resources. However, at present their safety can be guaranteed only on the two lowest rungs.

J. Perdew, K. Schmidt, AIP Conf. Proc. 577, 1 (2001).

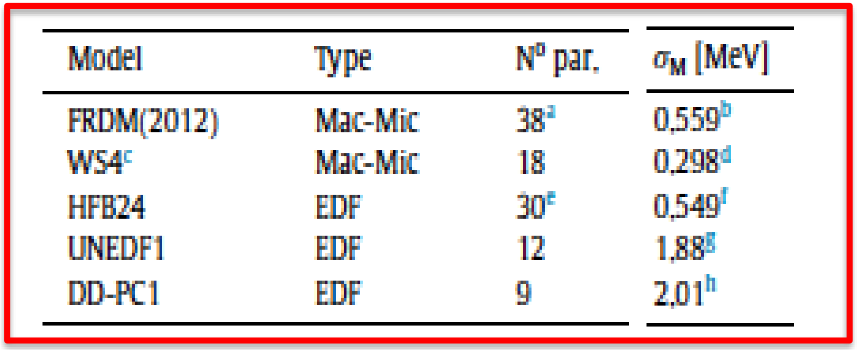
Error (th. vs. exp.) in the atomization energies of 20 molecules **[eV]**
(average of these energies **≈ 8.21 eV**)

LSDA	**GGA**	**mGGA**
1.4	**0.3**	**0.1**

Masses/binding energies of nuclei



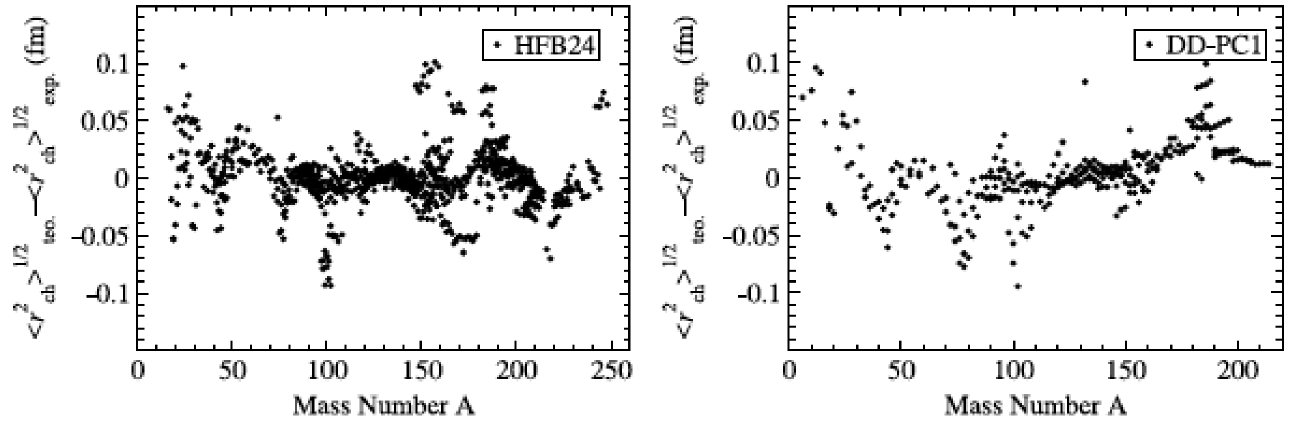
X. Roca-Maza and N.Paar, PPNP 101, 96 (2018)

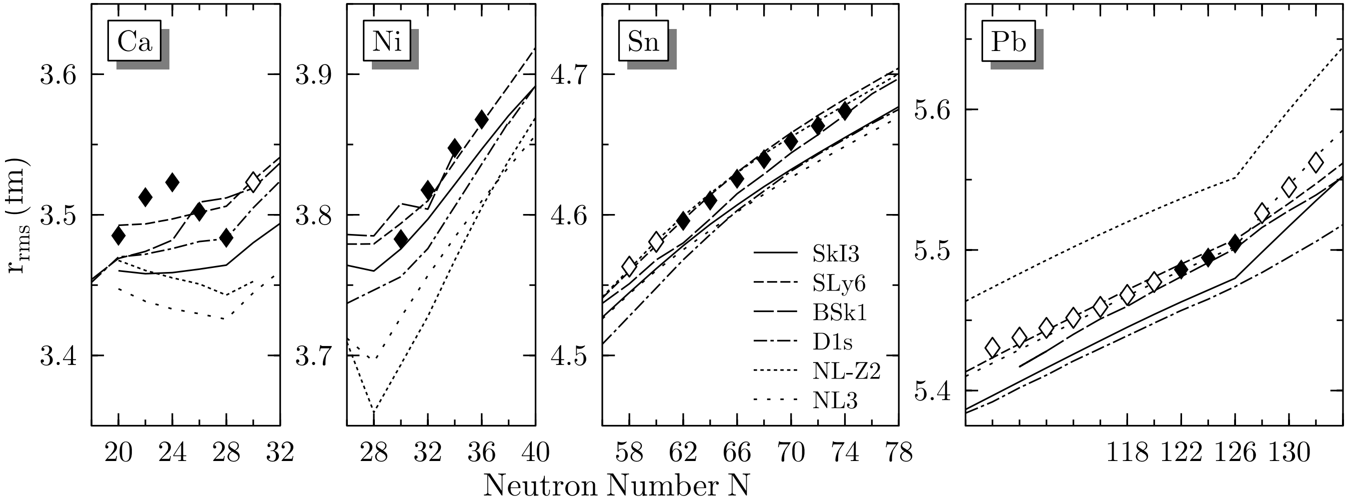


Model	Type	N° par.	σ_M [MeV]
FRDM(2012)	Mac-Mic	38[a]	0.559[b]
WS4[c]	Mac-Mic	18	0.298[d]
HFB24	EDF	30[e]	0.549[f]
UNEDF1	EDF	12	1.88[g]
DD-PC1	EDF	9	2.01[h]

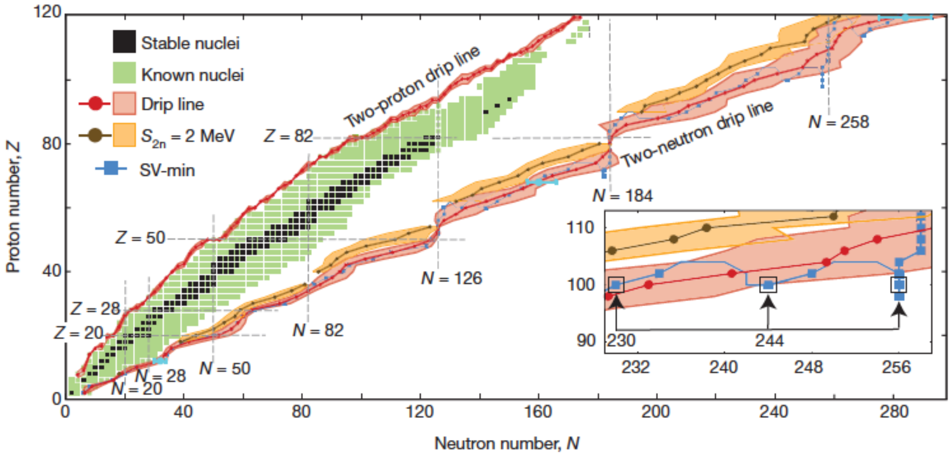
- Typical EDFs have errors ≈ MeV for the masses. 0.1 % to 1 % (!).
- More *ad hoc* parameters may push errors down (not necessarily the predictive power).
- Arches unavoidable?

Charge radii





Theoretical predictions of the drip lines



J. Erler *et al.*, Nature 486, 509 (2012) - SEDF

Evaluation of theoretical uncertainties.

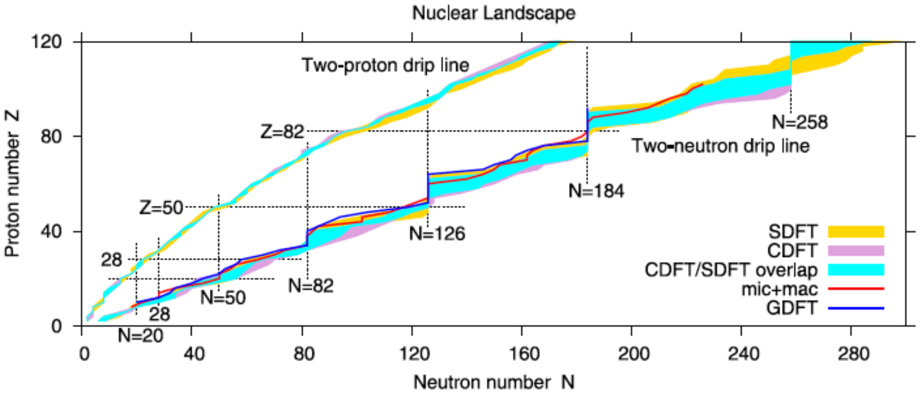
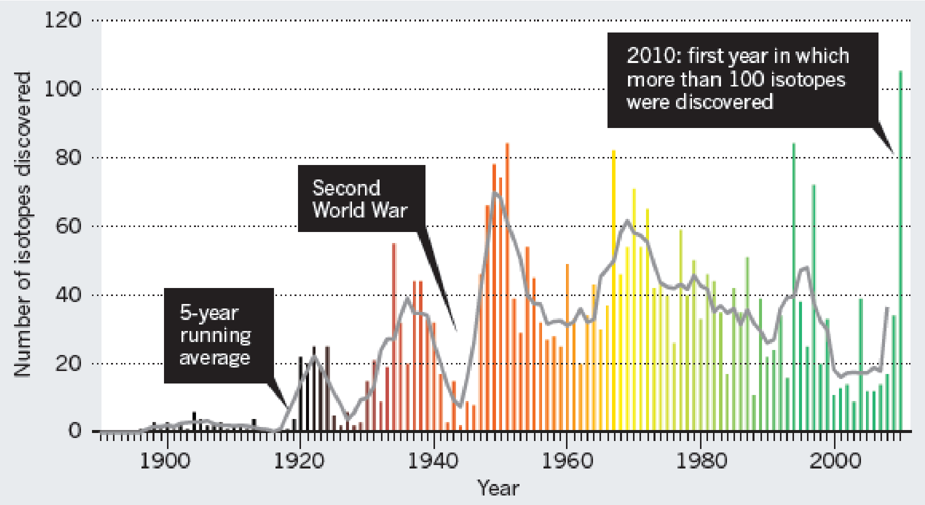


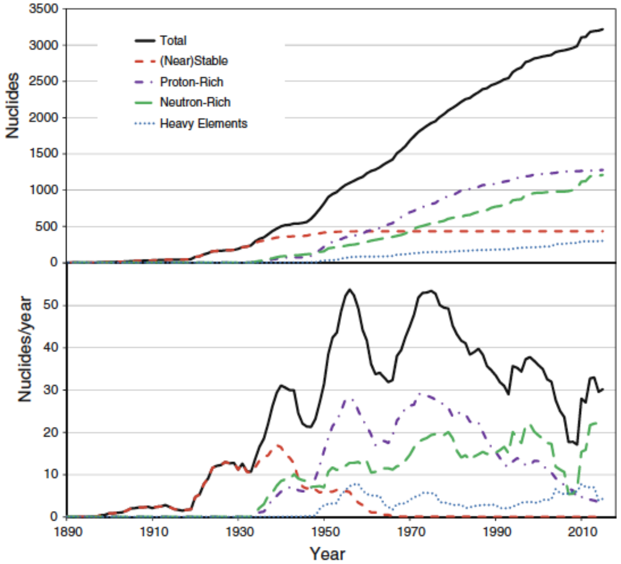
Fig. 4. The comparison of the uncertainties in the definition of two-proton and two-neutron drip lines obtained in CDFT and SDFT. The shaded areas are defined by the extremes of the predictions of the corresponding drip lines obtained with different parametrizations. The blue shaded area shows the area where the CDFT and SDFT results overlap. Non-overlapping regions are shown by dark yellow and plum colors for SDFT and CDFT, respectively. The results of the SDFT calculations are taken from the supplement to Ref. [2]. The two-neutron drip lines obtained by microscopic + macroscopic (FRDM [3]) and Gogny D1S DFT [5] calculations are shown by red and blue lines, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

A.V. Afanasjev *et al.*, Phys. Lett. B726, 680 (2013) - CEDF

The uncertainty on the drip lines position can increase up to about 10 mass units for large A.

Discovering new “exotic” isotopes



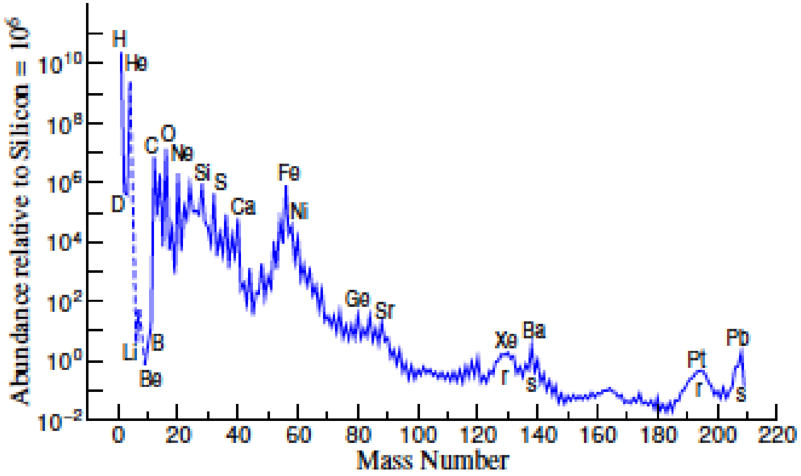


M. Thoennessen, B. Sherrill, Nature 473, 25 (2011) M. Thoennessen, Int. J. Mod. Phys. E 26, 1730003 (2017)

http://www.nscl.msu.edu/~thoennes/isotopes/.

Nuclear isotopes can be produced in the lab, **but how does or did Nature produce them ?**

The nucleosynthesis problem



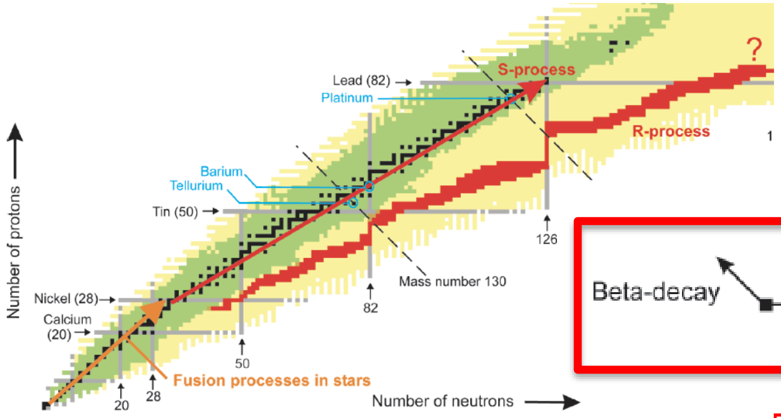

The 11 Greatest Unanswered Questions of Physics | DiscoverMagazine.com

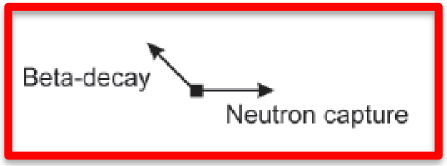
FROM THE FEBRUARY 2002 ISSUE

The 11 Greatest Unanswered Questions of Physics

How were the heavy elements from iron to uranium made?

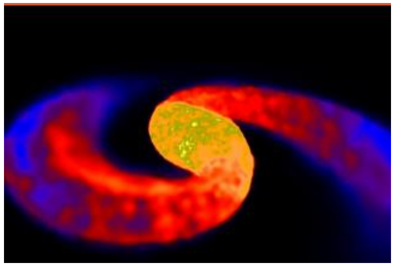
"How" means in which environment and by means of which nuclear processes





s-process and r-process.

Where does the r-process take place?

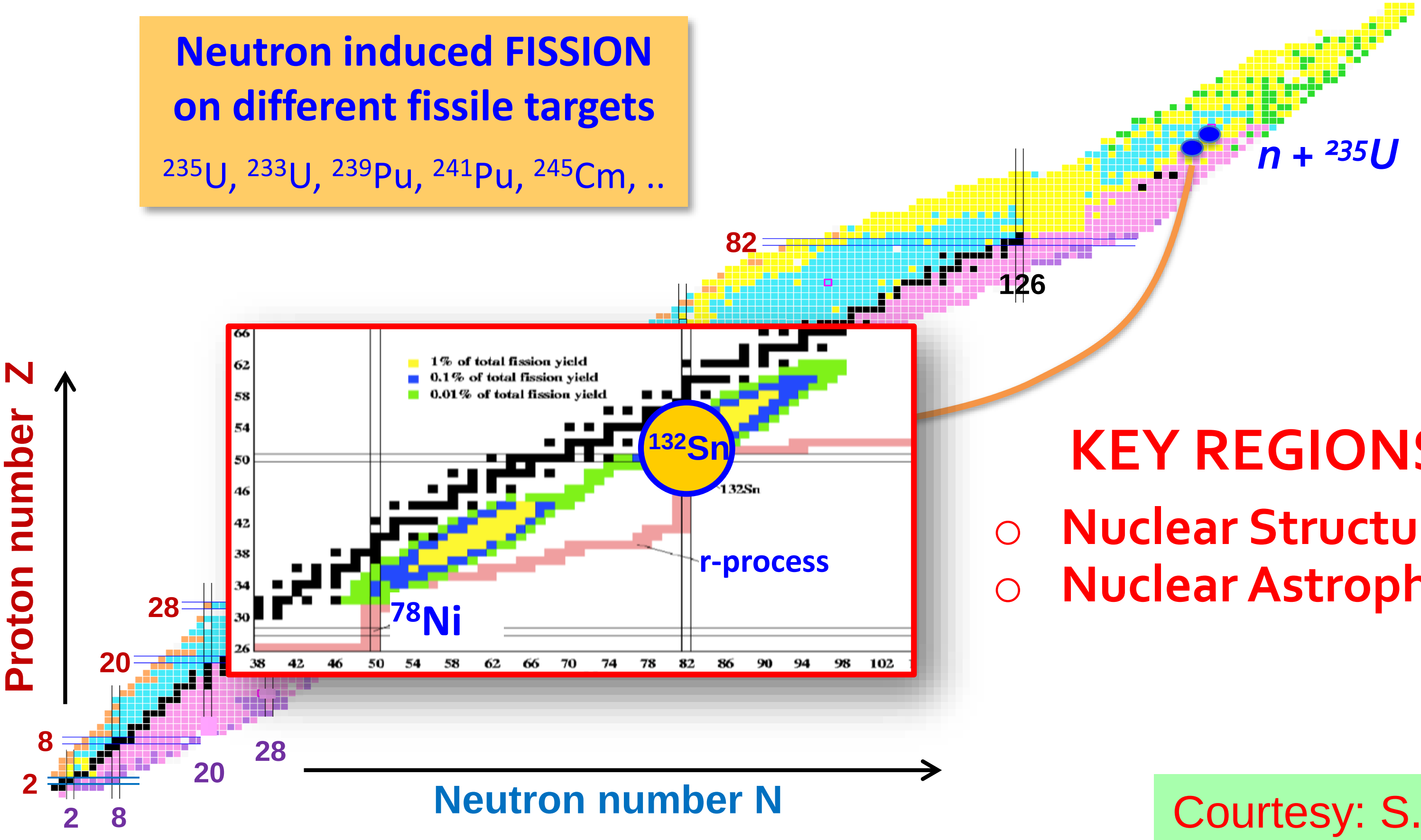


Figures: G. Martinez-Pinedo and H. Schatz

UNIQUE OPPORTUNITIES OFFERED BY ILL
INTENSE and very CLEAN thermal neutron beam

Neutron induced FISSION on different fissile targets

^{235}U, ^{233}U, ^{239}Pu, ^{241}Pu, ^{245}Cm, ..



KEY REGIONS for

- ○ Nuclear Structure
- ○ Nuclear Astrophysics

Courtesy: S. Leoni

The nuclear Equation of State (EoS): neutron stars, nuclear excitations, nuclear reactions

Nuclear matter

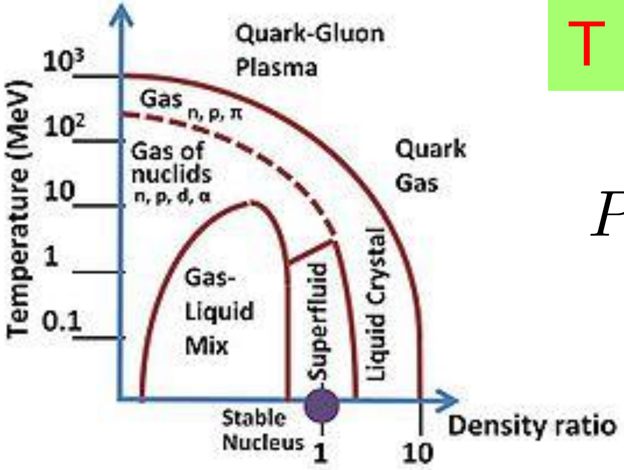
- Nuclear matter is an idealized UNIFORM system of neutrons and protons having constant density. (The Coulomb interaction among protons must be taken out).

- It is analogous to the uniform electron gas for condensed matter physicists.

WE STICK TO T = 0 !

$$\rho = \frac{A}{\Omega} \text{ with A fixed !}$$

$$P = -\frac{\partial E}{\partial \Omega} = -\frac{d\rho}{d\Omega}\frac{\partial E}{\partial \rho} = \frac{A}{\Omega^2}\frac{\partial E}{\partial \rho} = \rho^2 \frac{\partial}{\partial \rho}\frac{E}{A}$$



- We call **E/A the “equation of state” of nuclear matter.**

The EoS and the symmetry energy

Nuclear matter EOS

Symmetric matter EOS

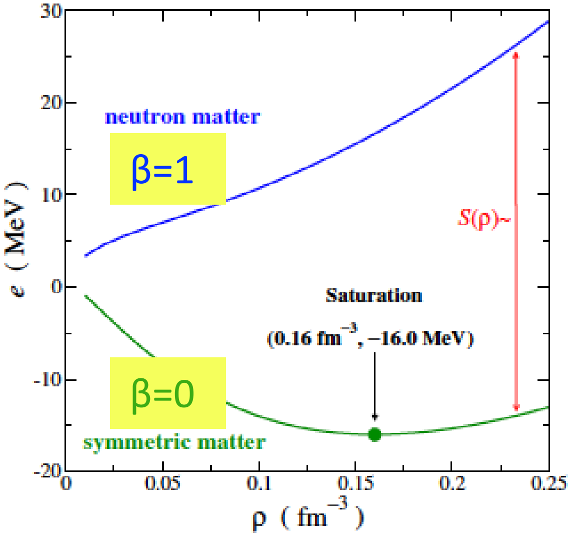
Symmetry energy S

$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta = 0) + S(\rho)\beta^2$$

$$\beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

$$S \equiv \frac{E}{A}(\text{neutron matter}) - \frac{E}{A}(\text{symmetric matter})$$

Expansion around
ρ_0= 0.16 fm^{-3}
SATURATION
POINT of SNM



$$\frac{E}{A}(\rho,\beta=0) = E_0 + \frac{1}{2}K_\infty\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \ldots$$

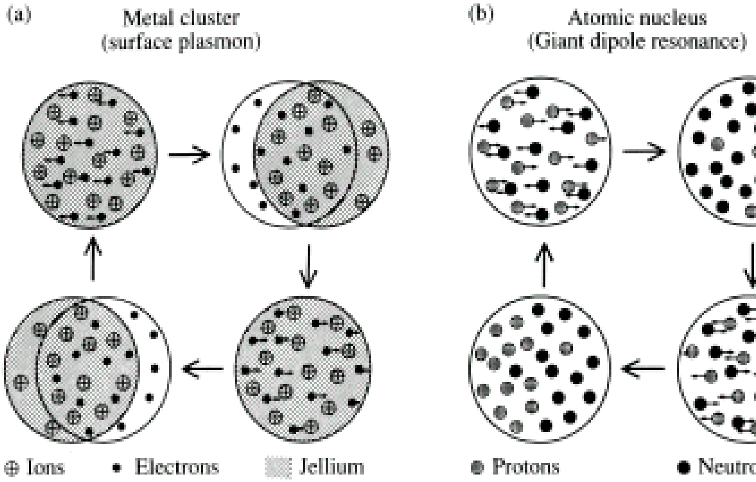
$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + + \frac{1}{2}K_{\rm sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \ldots$$

J = 31.6 ± 2.7 MeV
L = 58.9 ± 31.6 MeV
B.A. Li, 2016

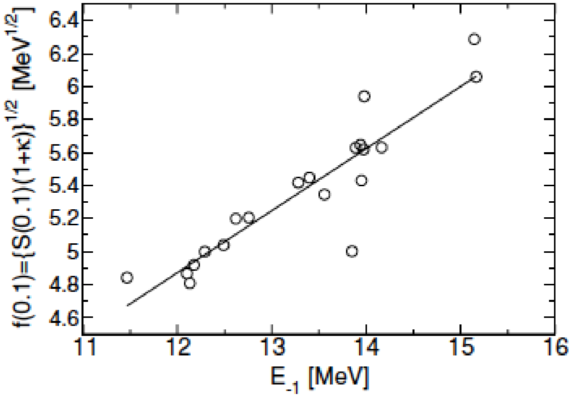
$$S(\rho_0) \equiv J$$
$$S'(\rho_0) \equiv L/3\rho_0$$
$$S''(\rho_0) \equiv K_{\rm sym}/9\rho_0^2$$

Isovector dipole collective motion

- **GDR** (Giant **Dipole Resonance**): excited by e.g. a photon beam impinging on a nucleus.

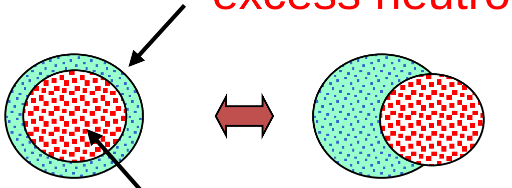


The larger the symmetry energy (energy per particle to change neutrons into protons), the higher the energy of this kind of oscillation modes



L. Trippa *et al.*, PRC 77, 061304(R) (2008)

- Neutron-rich nuclei can display other kinds of modes.



excess neutrons

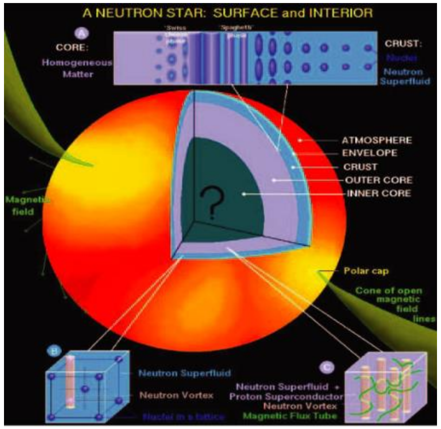
“core”

$$E_{\mathrm{IVGR}} \approx \sqrt{\frac{\partial^2 E}{\partial \beta^2}} \approx \sqrt{S(\rho)}$$

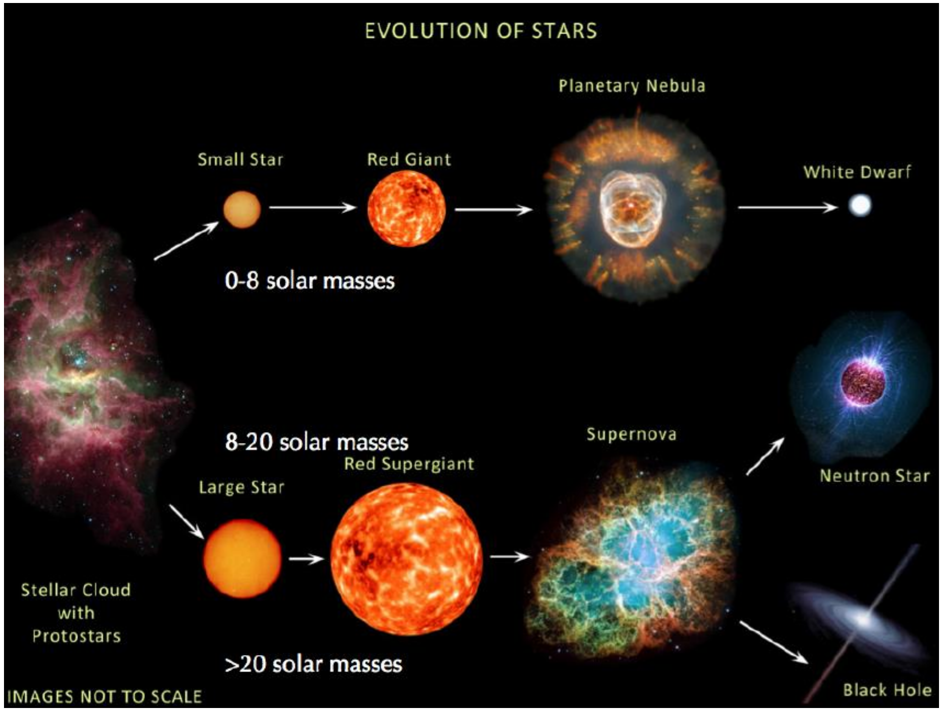
The existence of stars is determined by the **balance** between gravitational pressure and thermonuclear reactions.

If exothermic nuclear reactions are no longer possible, the star ends its life. The after-life destiny depends on the initial mass of the star.

Neutron stars: a possible outcome. Balance between gravitational pressure and energy of neutron matter.



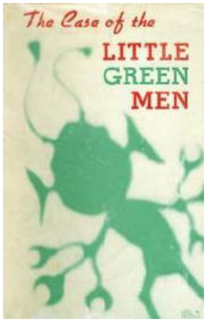
Courtesy: D. Page



Neutron stars

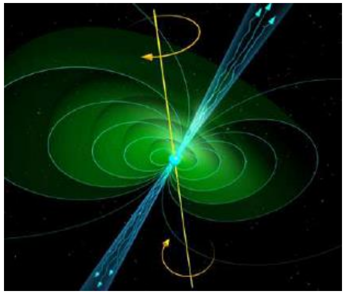
- Postulated by **Landau** after the neutron was discovered
http://www.ift.uni.wroc.pl/%7Ekarp44/talks/yakovlev.pdf

- Discovered many years later by **J. Bell and A. Hewish** (Nobel prize 1974)



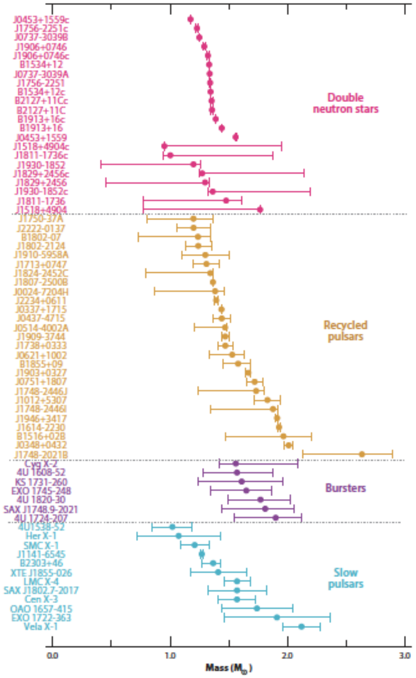
- Masses known much more precisely than radii

- Characterised by huge magnetic fields

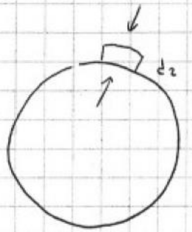


$$B \sim 10^{12}\ \mathrm{G}$$

F. Özel and P. Freire, ARAA 54, 401 (2016)



Tolman-Oppenheimer-Volkov equation



$$p(r+dr) = F(r+dr)/A$$

$$p(r) = F(r)/A$$

$$\frac{dp}{dr} = \frac{1}{A}\frac{dF}{dr} = \frac{1}{\cancel{4\pi r^2}}(-G)\frac{M(r)\overbrace{\cancel{4\pi r^2}\rho(r)\cancel{dr}}^{dm}}{r^2}\frac{1}{\cancel{dr}}$$

$$\frac{dp}{dr} = -G\frac{\rho(r)M(r)}{r^2} \quad ⊛$$

$$\frac{dM}{dr} = 4\pi r^2\rho(r) \quad , \quad M(r) = 4\pi\int_0^r dr' r'^2 \rho(r')$$

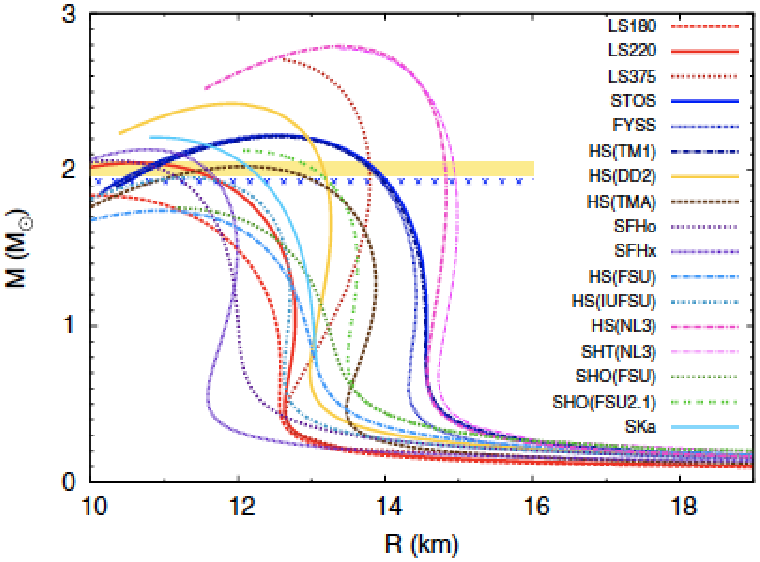
Classical gravity (Newton)

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2c^2}$$

General relativity corrections (TOV)

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2c^2}\frac{\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{\varepsilon(r)}\right)}{1-\frac{2Gm(r)}{rc^2}}$$

One has to input P(ρ) from the nuclear EoS and solve a simple differential equation.



M. Oertel, M. Hempel, T. Klähn, S. Typel, Rev. Mod. Phys. 89, 015007 (2017)

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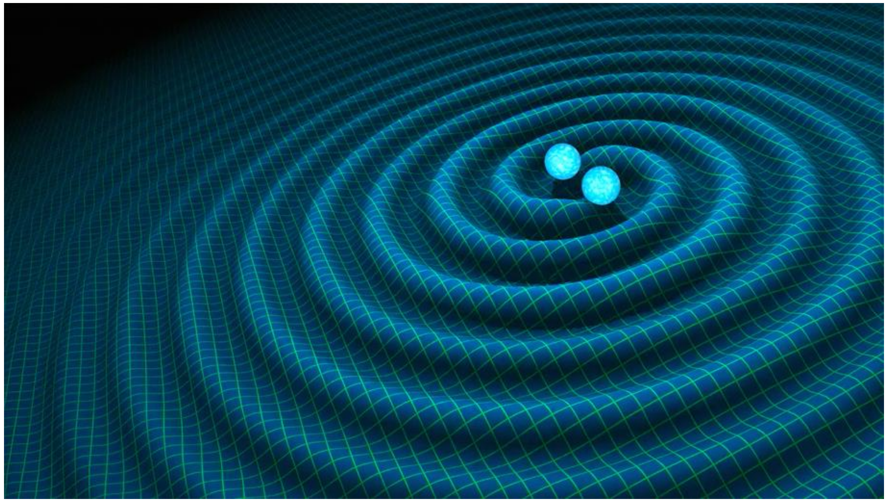
The merging of neutron stars

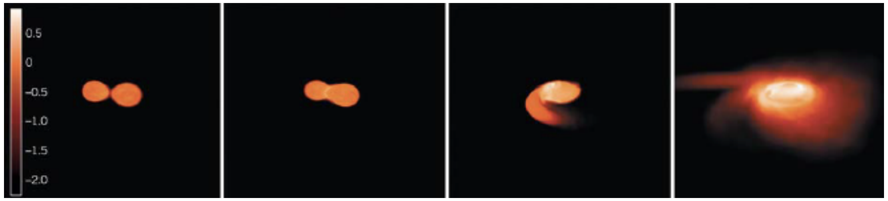
- 2015: first observation of Gravitational Waves (GW), awarded Nobel 2017.
- 17/8/2017 ”multi-messenger” observation of NS merging (GW, GRB, X-rays...).


GW170817 Press Release

LIGO and Virgo make first detection of gravitational waves produced by colliding neutron stars

B.P. Abbott et al., Astrophysical J. Lett. 848:L12 (2017)





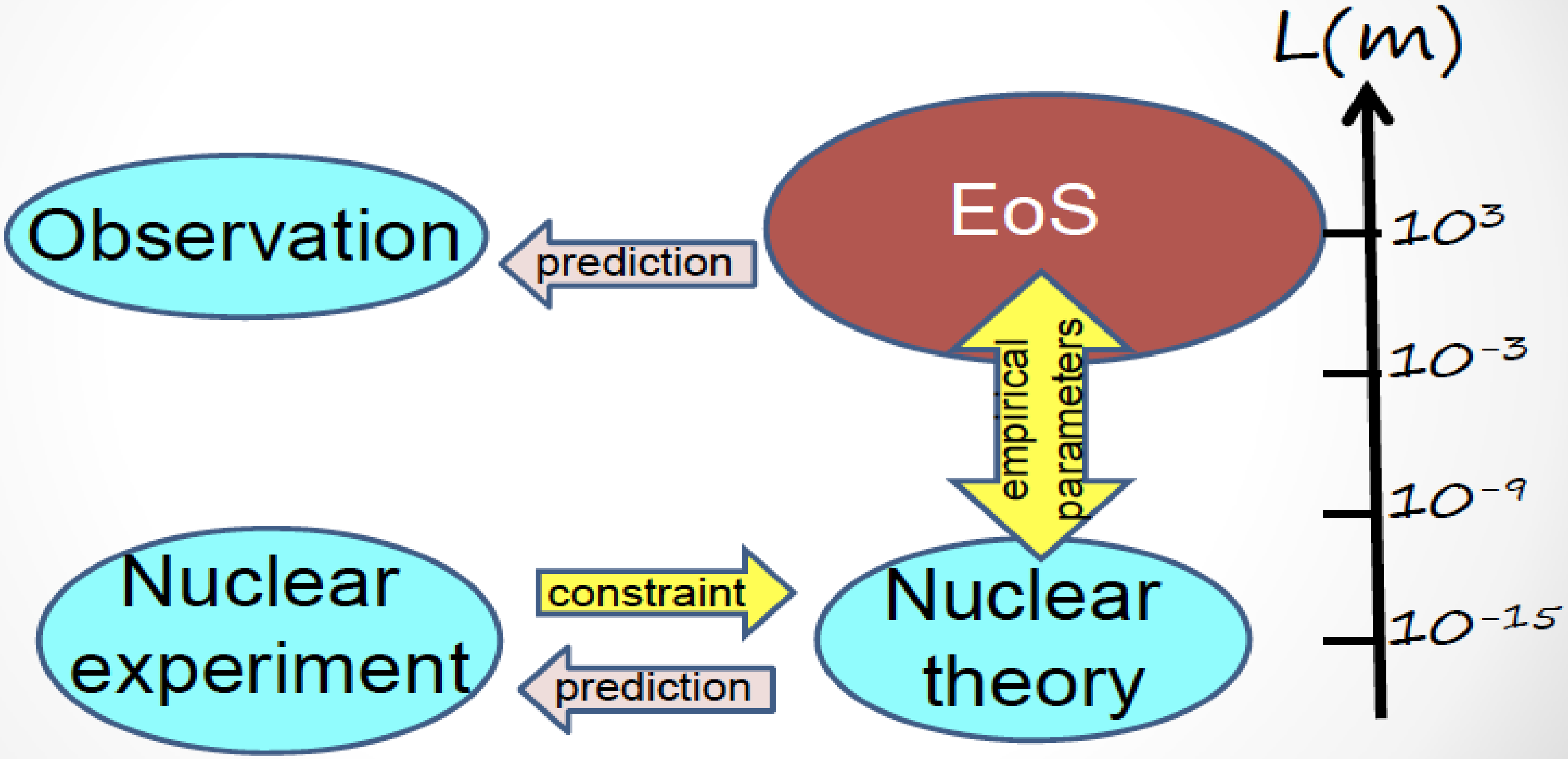
In the simulations tidal effects are quite relevant and they depend on the uncertainties on the nuclear EoS

$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 R}{GM} \right)^2$$

Nuclear experiments on isovector observables are relevant for the physics of GWs

Constraining the empirical parameters:

jumping across the scales!

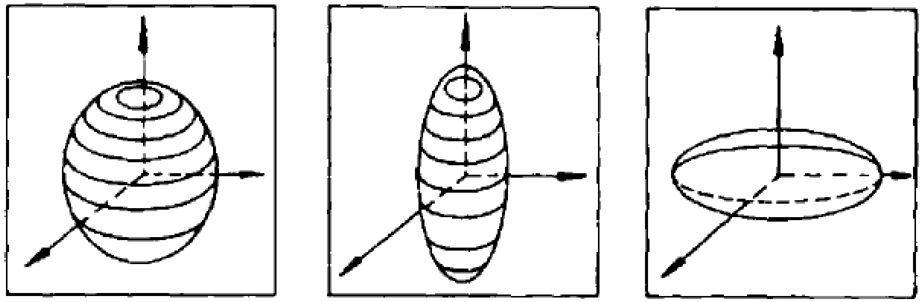


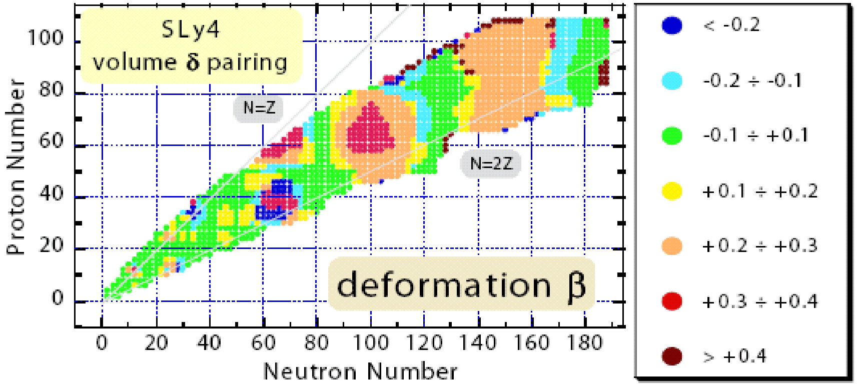
Courtesy: F. Gulminelli

Milano, April 2018

Nuclear correlations: coupling between single-particle and nuclear shapes

Nuclear deformation





In the case of axial quadrupole deformations:

$$\frac{\delta R}{R} \sim \beta$$

Intrinsic quadrupole moment $\langle Q \rangle \sim \beta$

The gain in energy due to deformation is correlated with the number of protons and neutrons outside closed shells.

There are non-axial deformations:

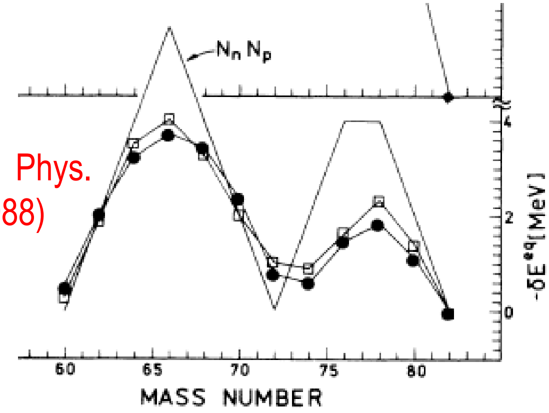
$$\delta R_x = R\left(\frac{\pi}{2}, 0\right) - R_0 = R_0\sqrt{\frac{5}{4\pi}}\beta cos\left(\gamma - \frac{2\pi}{3}\right)$$

$$\delta R_y = R\left(\frac{\pi}{2}, \frac{\pi}{2}\right) - R_0 = R_0\sqrt{\frac{5}{4\pi}}\beta cos\left(\gamma + \frac{2\pi}{3}\right)$$

$$\delta R_z = R\,(0,0) - R_0 = R_0\sqrt{\frac{5}{4\pi}}\beta cos\;\gamma$$

^{A}Ge

J. Dobaczewski et al., Phys. Rev. Lett. 60, 2254 (1988)



The nuclear Jahn-Teller effect

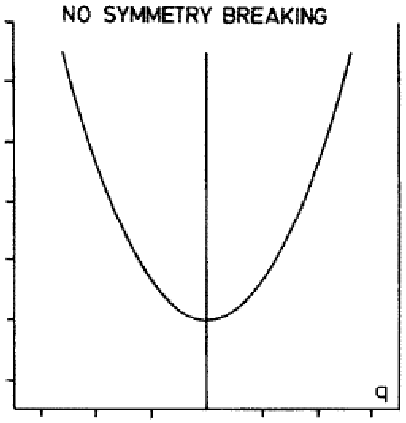

NO SYMMETRY BREAKING

Fig. 2a

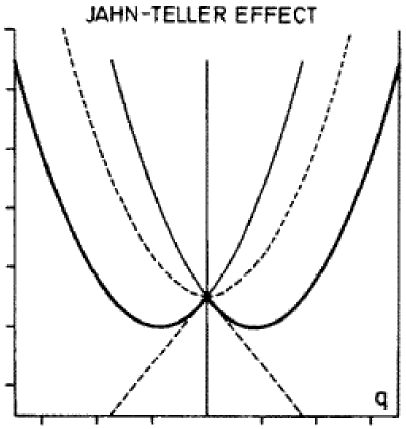

JAHN-TELLER EFFECT

Fig. 2b

In molecules, the coupling with vibrations breaks the degeneracy between electronic states.

In nuclei the **energy scale** is rather different: both **nucleons and shape vibrations are at the MeV scale**.

Nonetheless, in medium-heavy nuclei there are many, almost degenerate single-nucleon levels.

Coupling with shape vibrations can be strong.

In the case of quadrupole vibrations, this has been recognised long ago.

P.-G. Reinhard and E. W. Otten, Nucl. Phys. A 420, 173 (1984)

Octupole correlations/shapes

nature

GOING PEAR-SHAPED

Exotic radium-224 atomic nuclei seen at CERN

STAYING IN TOUCH

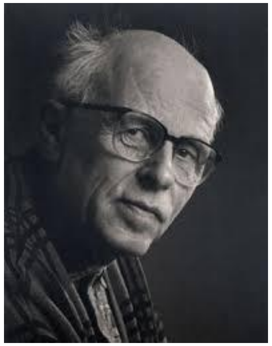
HEATED ARGUMENT

THE AGE OF THE BRAIN

^{224}Ra

Pear-shaped nuclei discovery challenges time travel hopes

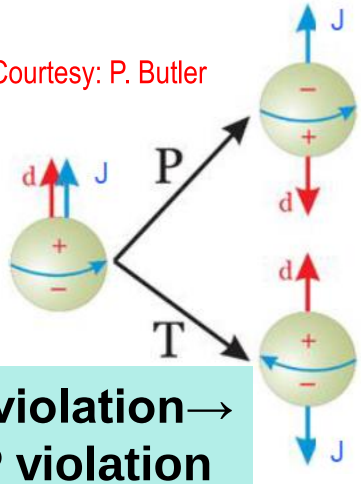
Tests of the standard model in nuclei



CP violation may be needed to explain matter-antimatter asymmetry. Cf. A. Sacharov.

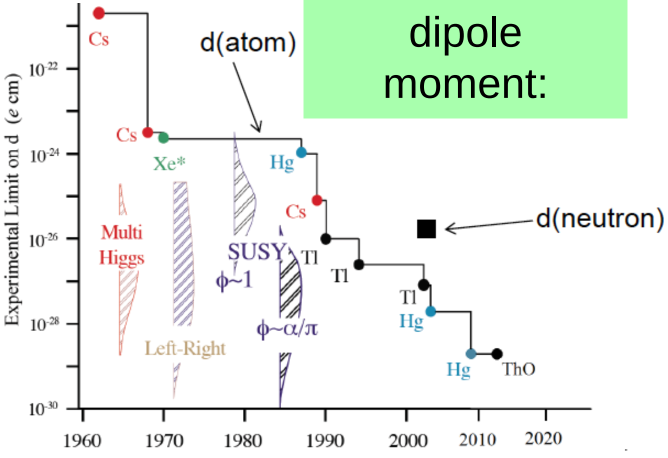
A static dipole moment violates CP:

Courtesy: P. Butler



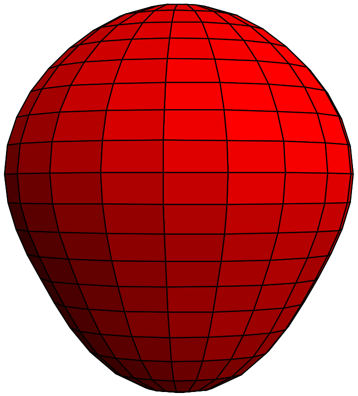
T violation→ CP violation

Searches for dipole moment:



Schiff:
If a nucleus is pointlike **its dipole moment cancels exactly the electronic dipole moment.** ☹

Pear-shape nucleus:
Something left (quenching may be around 10^{-3}).



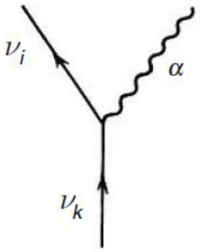
Particle-vibration coupling (PVC): general philosophy

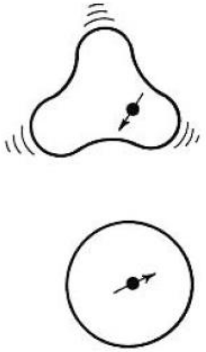
- The basic idea is that in spherical nuclei there are **single-particle states** and (mainly surface) **collective vibrations**. The spectra result from their **interplay.** (Deformed nuclei: particle-rotation coupling).

- **Vibrations = phonons.**

- Odd nuclei:

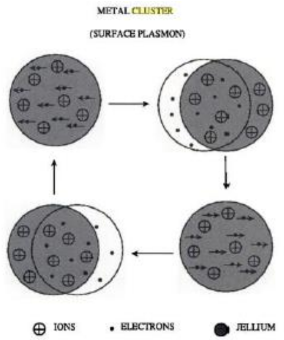
core + 1 particle +
1 particle plus phonon …

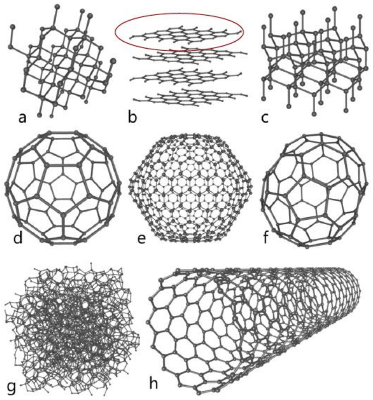




Electron-plasmon coupling

Phonons and Plasmons

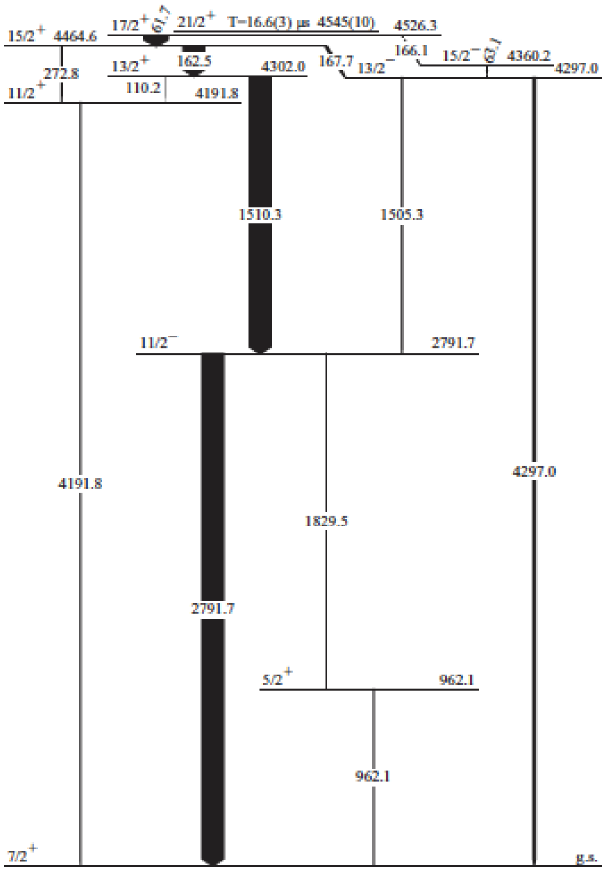




(clusters, fullerenes)

Despite the importance of the region around ^{132}Sn, the information about **low-lying states of neighbouring nuclei** need still be completed.

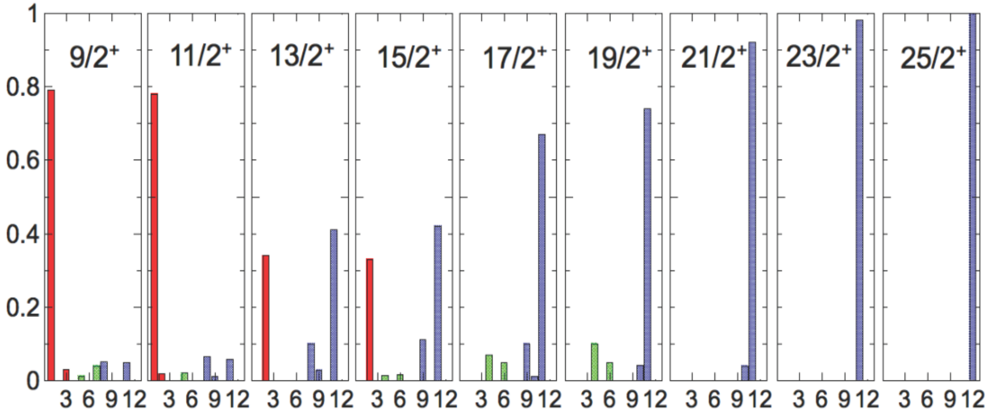
^{133}Sb



- Recently new measurements (G. Bocchi *et al.*) have shed light on some **HIGHER SPIN** states (up tp 25/2$^+$) in ^{133}Sb.

  B(M1, 15/2$^+$→13/2$^+$) = 0.24 W.u.
  B(M1, 13/2$^+$→11/2$^+$) = 0.004 W.u.
  (ratio = 60).

- **We have been able to explain this sudden change in a model that takes into account the aforementioned couplings and account for the change of the wave function of the states.**



red: $\pi g_{9/2} \otimes 2^+, 4^+$

blue: $\pi g_{9/2}\ \nu h^{-1}_{11/2}\ \nu f_{7/2}$

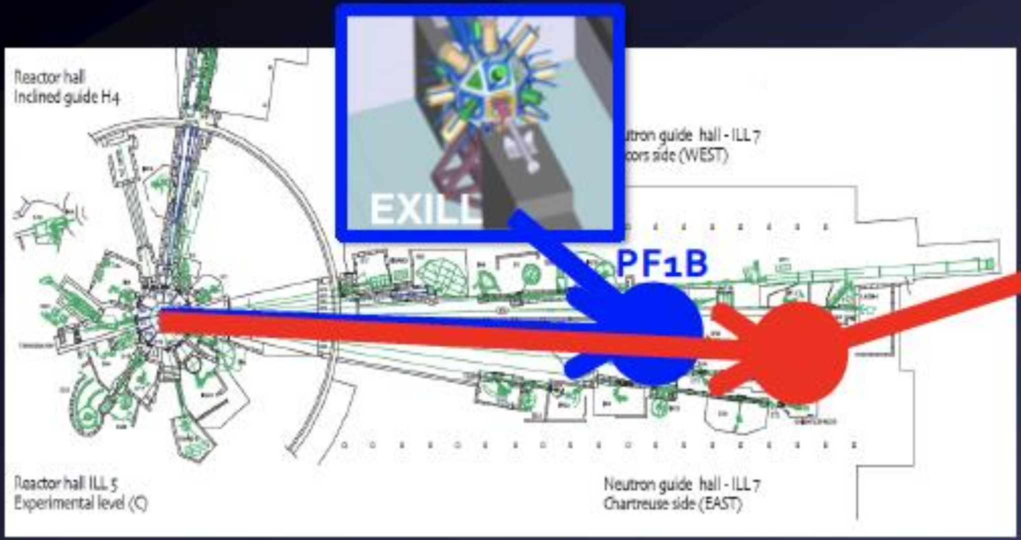
This nature is reflected in the M1 transition probabilities.

The wave functions of 15/2$^+$ and 13/2$^+$ are dominated by $g_{9/2}$, $h_{11/2}^{-1}$ $f_{7/2}$, so the B(M1) transition is made up with s.p. amplitudes. $B(M1)_{th}$ = 0.021 W.u.

In the case of the transition 13/2$^+$ → 11/2$^+$, the final state has phonon component so there is a mismatch in the components and B(M1) is quenched, $B(M1)_{th}$ = 0.001 W.u. Ratio = 20.

Competitive Instrumentation for GAMMA Spectroscopy

Large Germanium Arrays



The EXILL Campaign (2012-2013)

2 Reactor Cycles ≈ 100 days > 95% DATA TAKING

The FIPPS Permanent Setup (since 2016)

International Collaboration with 18 Institutions

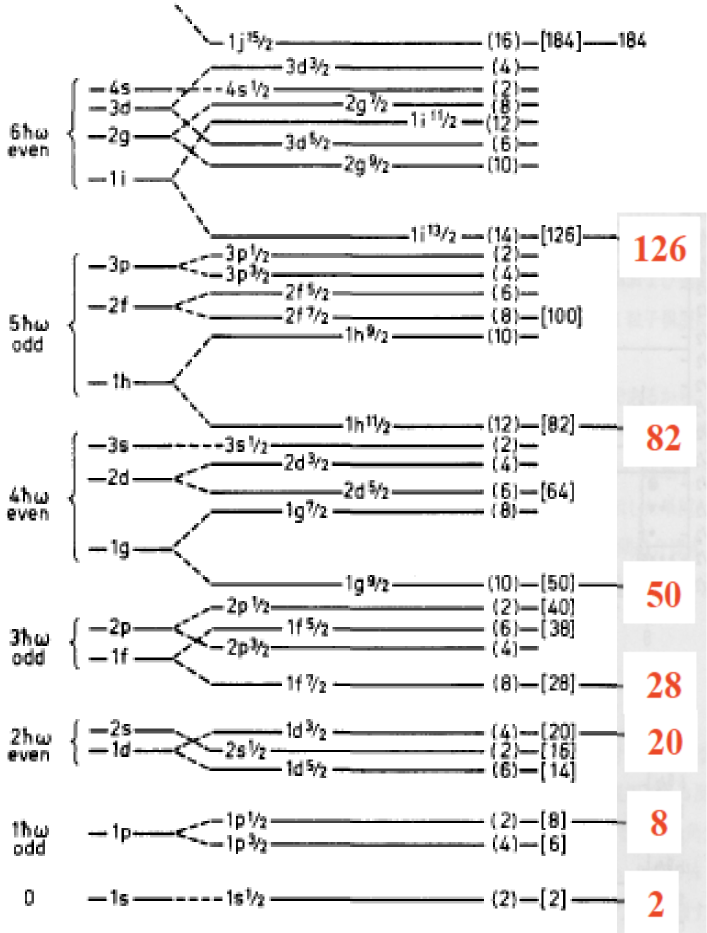
Courtesy: S. Leoni

Thank you!

Backup slides

The nuclear independent particle model

≈ MeV (levels from about -50 MeV)



Many experimental evidences point to the fact that nucleons move in nuclei, to a first approximation, as **independent** particles.

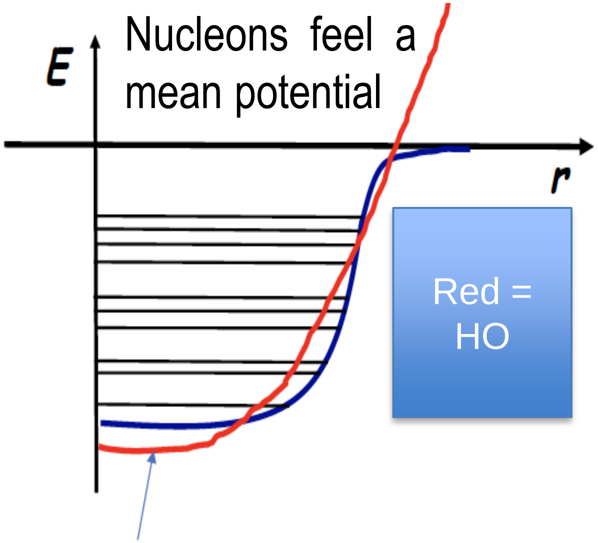
Examples: evidence of shells, ground-state of nuclei around closed shells (^{17}O with Z=8, N=9 has $J^{\pi}=5/2^{+}$) …

M.G. Mayer,
J.H.D. Jensen

From mean-field approximation to DFT

Strategies:

1. Start from a two-body (three-body) effective force and define H = T + V_{eff} [ϱ]. Write the expectation value on a general Slater determinant.
   “DFT is an exactification of Hartree-Fock” (W. Kohn).
2. Write directly the Energy Density Functional (EDF).



$$E = \langle \Psi | \hat{H} | \Psi \rangle \approx \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}]$$

$|\Phi\rangle$ **Slater determinant**

The Hohenberg-Kohn theorem (HK)

The original theorem and its proof can be found in P. Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)[1]. We have in mind a system of interacting fermions ($H = T + V$) in some external potential V_{ext}.

a) There exist a functional of the fermion density

$$E_{V_{\text{ext}}}[\rho] = \langle\Psi|T + V + V_{\text{ext}}|\Psi\rangle = F[\rho] + \int d^3r\ V_{\text{ext}}(r)\rho(r)$$

and the part denoted by F is universal (for nuclei, it would be the only part).

b) It holds:

$$\boxed{\min_\Psi\ \langle\Psi|T + V + V_{\text{ext}}|\Psi\rangle = \min_\rho\ E_{V_{\text{ext}}}[\rho]}$$

or, in other words, at least in principle this functional has a minimum at the exact ground-state density where it assumes the exact energy as a value.

[1] More than 20,000 citations

A realization: the Kohn-Sham scheme

We assume that the density can be expressed in terms of **single-particle orbitals**, and that the kinetic energy has the simple form:

$$\rho(\vec{r}) = \sum_i \phi^*(\vec{r})\phi_i(\vec{r}) \qquad T = \sum_i \int d^3r\ \phi_i^*(\vec{r}) \left(-\frac{\hbar^2\nabla_i^2}{2m}\right) \phi_i(\vec{r})$$

We have said that the energy must be minimized, but we add a constraint associated with the fact that we want **orbitals that form an orthonormal set** (Lagrange multiplier)

$$E - \sum_i \varepsilon_i \int d^3r\ \phi_i^*(\vec{r})\phi_i(\vec{r}) = T + F[\rho] + \int d^3r\ V_{\rm ext}(\vec{r})\rho(\vec{r}) - \sum_i \varepsilon_i \int d^3r\ \phi_i^*(\vec{r})\phi_i(\vec{r})$$

The variation of this quantity, $\delta(\ldots) = 0$ produces a Schrödinger-like equation:

$$\boxed{\left(-\frac{\hbar^2\nabla_i^2}{2m} + \frac{\delta F}{\delta\rho} + V_{\rm ext}\right)\phi_i(\vec{r}) = \varepsilon_i\phi_i(\vec{r})}$$

$$\boxed{h\phi_i = \varepsilon_i\phi_i}$$

The good, the bad and the ugly !

- The HK theorem has been **generalised** in (almost all) possible ways: degenerate ground-state, magnetic systems (**m** or spin densities), finite temperature, relativistic case … (the list might be not exhaustive).

- **In all cases one then build the energy functional E and tries to find a minimum.**

- Kohn-Sham is a widely used scheme.

- Several algorithms.

One must use the functional derivative:

$$F[\rho] = \int d^3r \; \rho(r) f(r) \Rightarrow \frac{\delta F}{\delta \rho(r)} = f(r)$$

- Then, all quantities are in principle given (!).

- **The REAL weak point is that the various proofs of HK theorems do not give any clue on HOW to build the functional *F*.**

- **<u>There are many realizations of such theory and no obvious route for systematic improvement.</u>**

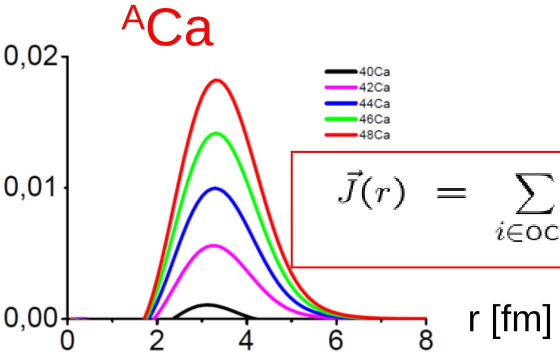
Realistic functionals

- One has to consider **several possible densities** (including spin densities, isospin densities, gradients …).

$$\vec{\nabla}\rho$$

$$\tau = \sum_i |\vec{\nabla}\phi_i|^2$$

$$\vec{J} = \sum_{i,\sigma,\sigma'} \phi_i^\dagger \vec{\nabla}\phi_i \times \langle \sigma' | \vec{\sigma} | \sigma \rangle$$



$$\vec{J}(r) = \sum_{i \in \mathrm{occ}} 2\,(2j_i + 1)\,\langle\, \vec{\ell} \cdot \vec{s}\,\rangle \frac{\vec{r}}{r^4} u_i^2(r)$$

- **Functionals will include all possible scalars made up with these quantities that respect the <u>symmetries</u> of the systems.**

(translations, Galilean boosts, rotations, parity and time-reversal)

- There exist relativistic functionals.

The Skyrme force

attraction

$$\hat{v}_{\mathrm{Sk}}(r_{12}) = \underline{t_0(1+x_0\hat{P}_\sigma)\delta(r_{12})} + \frac{1}{2}\underline{t_1(1+x_1\hat{P}_\sigma)(\hat{k}^{\dagger 2}\delta(r_{12})+\delta(r_{12})\hat{k}^2)}$$

$$\underline{+t_2(1+x_2\hat{P}_\sigma)\hat{k}^\dagger\cdot\delta(r_{12})\hat{k}} + \underline{\frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\delta(r_{12})\rho^\alpha\left(\frac{r_1+r_2}{2}\right)}$$

$$+\mathrm{i}W_0(\hat{\sigma}_1+\hat{\sigma}_2)\cdot\hat{k}^\dagger\times\delta(r_{12})\hat{k}.$$

short-range repulsion

$$k = \frac{i}{2}\left(\vec{\nabla}_1 - \vec{\nabla}_2\right)$$

- There are velocity-dependent terms which mimic the finite-range. They are related to m*.

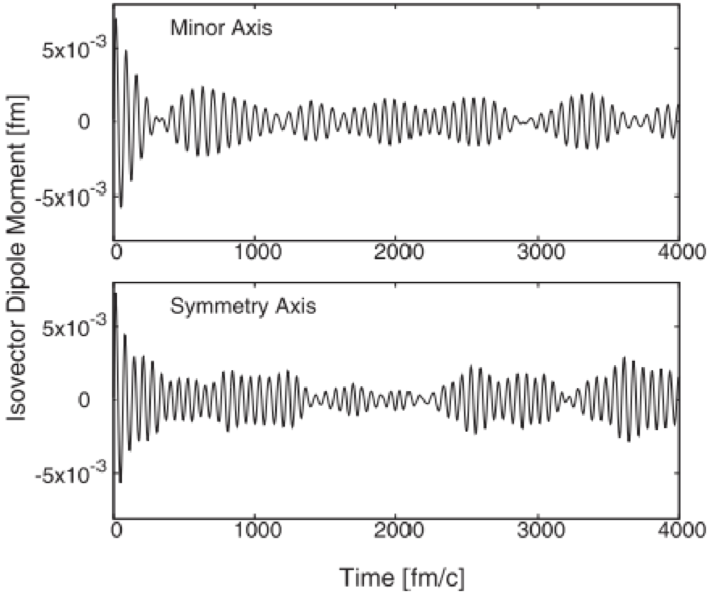
- The last term is a zero-range spin-orbit.

- In total: 10 free parameters to be fitted (typically).

Time-dependent DFT

$$\boxed{h\phi_i = \varepsilon_i \phi_i}$$

From: P. Stevenson (U. Surrey)



The FT will give the energy of the mode(s).

In the time-dependent case, one can solve the evolution equation for the density directly:

$$h(t) = h + f(t) \qquad [h(t), \rho(t)] = i\hbar\, \dot{\rho}(t)$$

$$\rho(t=0) \neq \rho_{\mathrm{g.s.}}$$

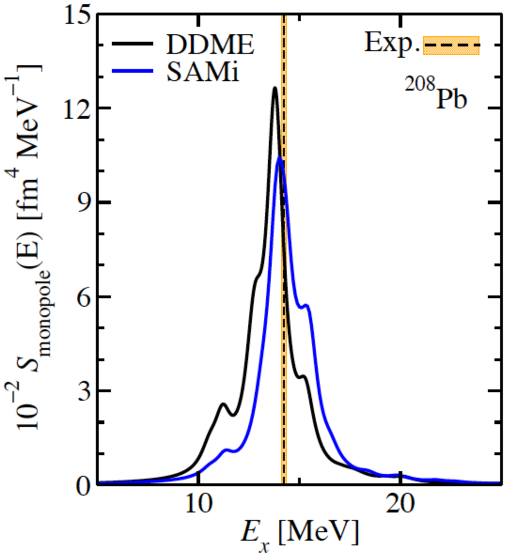
$$\rho(t=\Delta t) = U(t=0, t=\Delta t)\rho(t=0) \qquad U = e^{-i\frac{\Delta t}{\hbar}\cdot h}$$

Often the equation for the density is linearized and solved on a basis (**Random Phase Approximation or RPA**).

Università degli Studi
and INFN, MIlano

The Giant Monopole Resonance and EoS

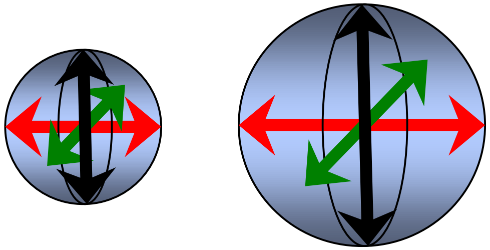
One further example of study which is not carried out for mere “academic purposes”, but to shed light on **more general properties of nature**…



SN1987a

Breathing mode: in this case its energy is correlated with the compressibility of nuclear matter.

$$\chi \equiv -\frac{1}{\Omega}\left(\frac{\partial P}{\partial \Omega}\right)^{-1}$$



We better consider the density as a variable.

$$\rho = \frac{A}{\Omega}$$

Incompressibility:

$$\chi^{-1} = \rho^3 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right)$$

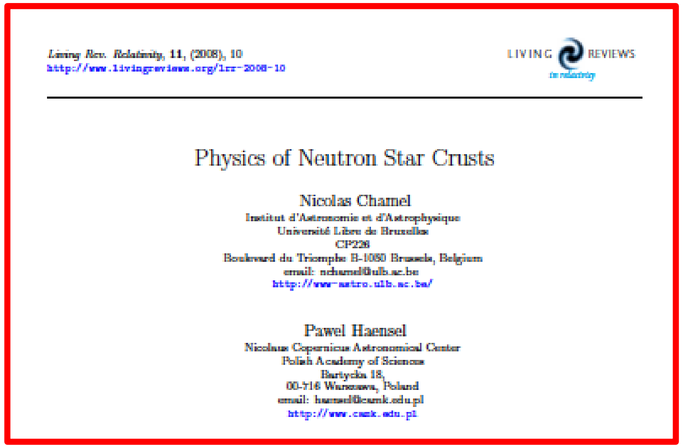
$$K_\infty = 9\rho_0^2 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right)_{\rho=\rho_0}$$

(**around 240 MeV**)

Neutron stars

$$B(N,Z) = a_V A - \cancel{a_S A^{2/3}} - a_A \frac{(N-Z)^2}{A} - \cancel{a_C \frac{Z^2}{A^{1/3}}} \cancel{+\delta} \begin{array}{l} + \frac{3}{5}\frac{GM^2}{R} \\ + \frac{3}{5}\frac{Gm^2A^2}{r_0 A^{1/3}} \end{array}$$

- A bound system of neutrons can exist due to the gravitational force if the number of neutrons is large enough (≈10^{55}-10^{56}).
- Neutron stars observed much later than they were postulated.
- Now some properties are observed (masses and, to a lesser extent: radii, magnetic field, thermal properties…).



Living Rev. Relativity, 11, (2008), 10
http://www.livingreviews.org/lrr-2008-10

LIVING REVIEWS
in relativity

Physics of Neutron Star Crusts

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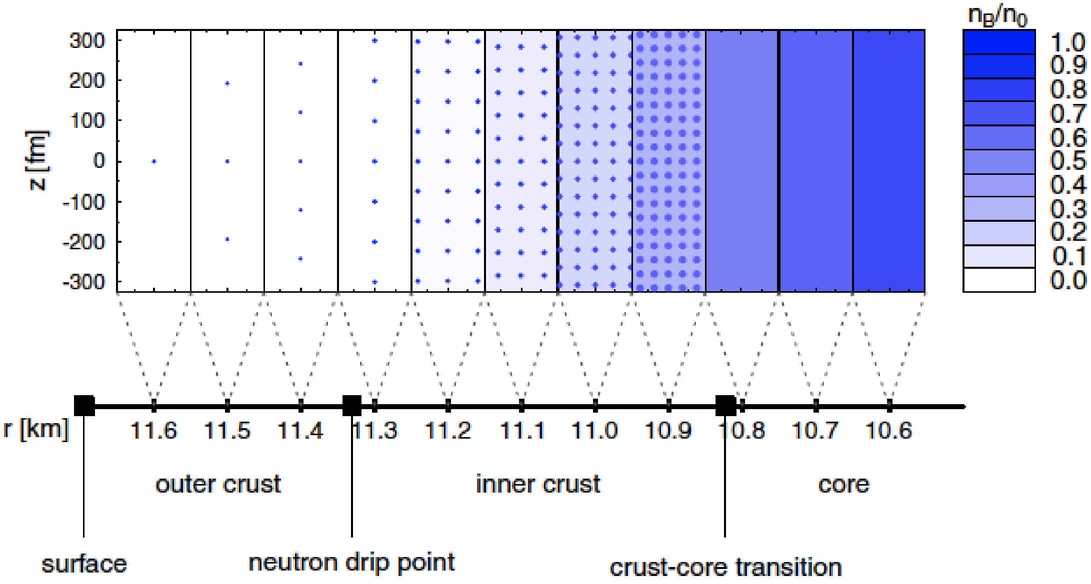
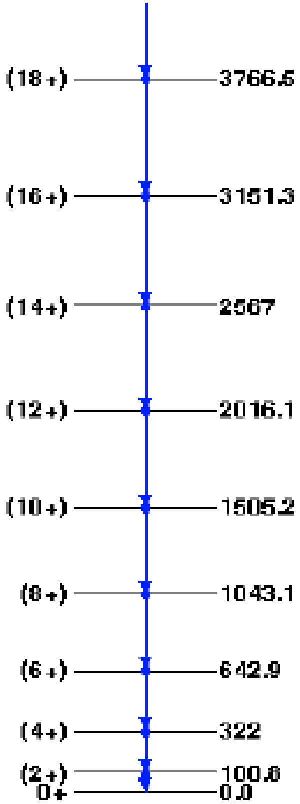


M. Oertel, *et al.*, Rev. Mod. Phys. 89, 015007 (2017)

Symmetry breaking and restoration



$$^{170}_{82}\mathrm{Hf}$$

$$E = \frac{\hbar^2}{2\mathcal{I}} J(J+1)$$

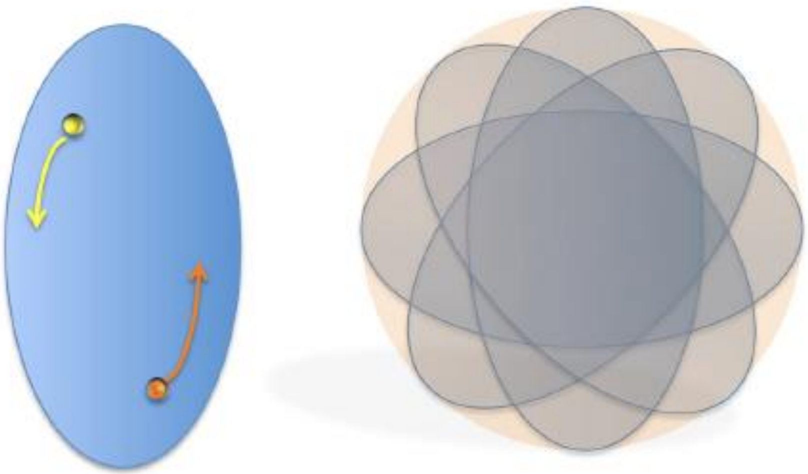
$$\tau_{\mathrm{SSB}} \sim \frac{\hbar}{E}$$

$$\hbar = 6.6\ 10^{-22}\ \mathrm{MeV} \cdot \mathrm{s}$$

Rotational motion restores the symmetry in the lab system.

This may happen on a time scale which is significantly larger than

$\tau_F \sim 10^{-22}$ s



(a) $\tau \approx \tau_F < \tau_{SSB}$ (b) $\tau > \tau_{SSB}$

From: T. Nakatsukasa