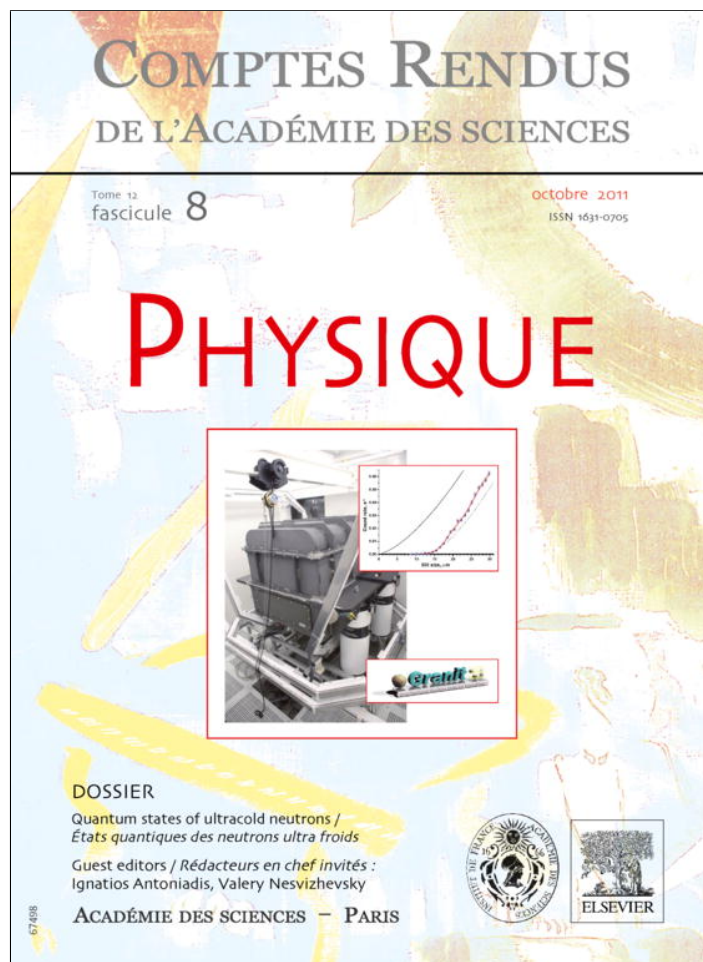


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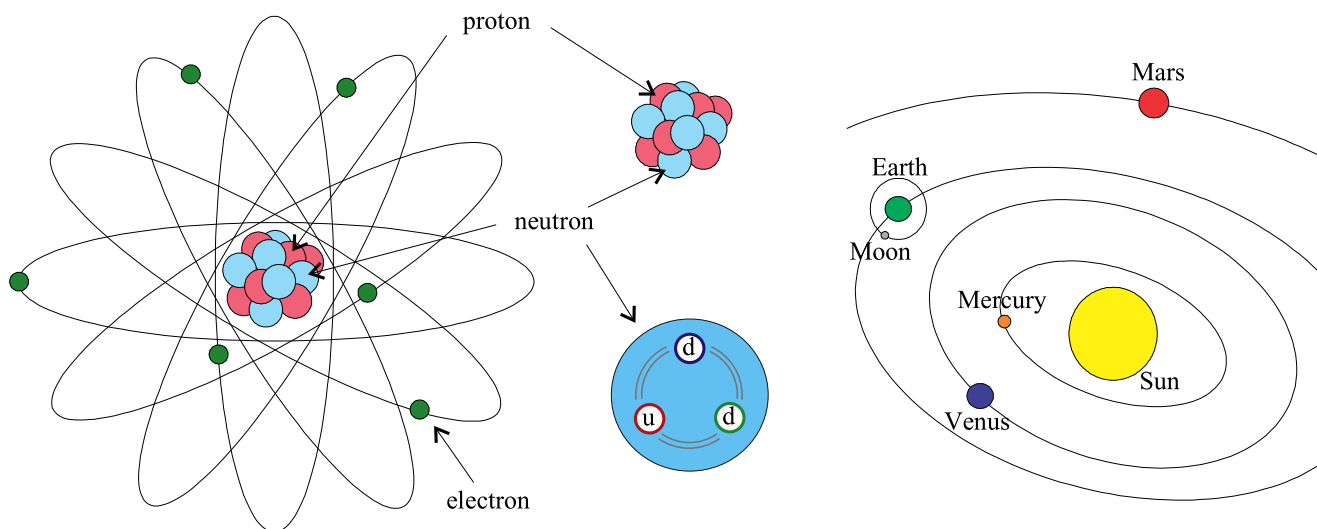


Ultra cold neutron quantum states / États quantiques des neutrons ultra froids

## Workshop GRANIT-2010, 14–19 February 2010, Les Houches, France

The workshop GRANIT-2010<sup>1</sup> on experimental and theoretical approaches to neutron quantum states in the gravitational field took place in Les Houches, France, on 14–19 February 2010. Its aim was to discuss the scientific program for the new GRANIT spectrometer in a broad context. The behavior of ultracold neutrons (UCN) in the gravitational field in GRANIT and in alternative experiments was presented, as well as applications of this phenomenon/spectrometer to various domains of physics, ranging from constraints for short-range interactions to neutron quantum optics and reflectometry using UCN. Topics were the scientific issues, instrumental and methodical developments, alternative methods, and new ideas. This is a follow-up for the GRANIT-2006<sup>2</sup> workshop, and the preceding annual meetings of our collaboration on the measurement of neutron quantum states in the gravitational field.

Fig. 1 shows bound states due to interactions of different types. A characterization of bound states has always served not only to explore the system under study, but also to better understand the interaction responsible for the binding force. The carbon atom is bound due to electromagnetic interaction. The carbon nucleus, as well as the nucleons in it, is bound due to strong interaction; nucleons are composed of quarks and gluons. The Solar System is bound due to gravitational interaction. A difference between the bound state “Solar System” and the other bound states is that the others need a quantum mechanical description while the Solar System does not. The reason is the size of possible gravitationally bound states, or the smallness of the gravitational interaction. Hence, only recently the first gravitationally bound quantum states were discovered.



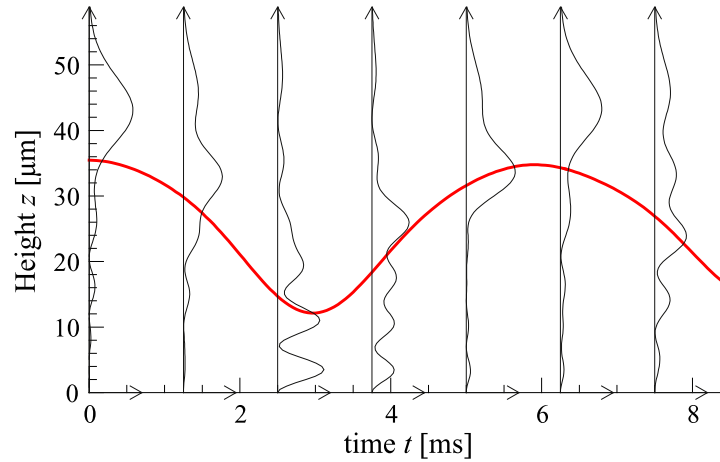
**Fig. 1.** Examples of bound states in Nature: simple model picture for a carbon atom, a carbon nucleus, a neutron, the (inner part of) the Solar System.

**Fig. 1.** Exemples d'états liés existants dans la Nature : modèle simple d'un atome de carbone, noyau de carbone, neutron, (partie interne du) Système Solaire.

For this discovery, one used the fact that ultracold neutrons (UCN) experience a neutron-optical potential (sometimes called Fermi potential) in matter of a size big enough that matter can totally reflect UCN if their energy is sufficiently low.

<sup>1</sup> <http://lpsc.in2p3.fr/Indico/conferenceDisplay.py?confId=371>.

<sup>2</sup> <http://lpsc.in2p3.fr/congres/granit06/index.php>.



**Fig. 2.** Evolution of the expectation value of the particle height (thick red line) and the square of the wave function (thin gray inserts, on the horizontal axis) with time for a neutron assumed to be a point particle which bounces on a horizontal mirror. At  $t = 0$ , the neutron is assumed to be in the initial quantum state  $\Psi = (\Psi_3 + \Psi_4 + \Psi_5 + \Psi_6)/2$  (we will define the notation in the next paragraph). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Fig. 2.** Evolution de la valeur moyenne de l'altitude de la particule (épaisse ligne rouge) et du carré de la fonction d'onde (lignes grises minces) en fonction du temps pour un neutron traité comme une particule ponctuelle qui rebondit sur un miroir horizontal. A  $t = 0$ , le neutron est dans l'état quantique initial  $\Psi = (\Psi_3 + \Psi_4 + \Psi_5 + \Psi_6)/2$  (les notations sont définies dans le prochain paragraphe).

In fact, the term “ultracold neutron” is loosely defined as the neutrons which are totally reflected at all incidence angles by the matter used for the confining bottle for them. Here, we focus on UCN with even lower energy, for which we can approximate the potential in matter as infinite. These neutrons bounce on a horizontal table essentially like ping-pong balls on a ping-pong table. They are confined in a potential which is linearly rising with height above the table due to gravitation, and which is infinite below. In order to detect effects of the quantization, one has to focus on neutrons in one of the lowest possible states.

In Fig. 2, we show a point particle that is free falling between bounces on a horizontal mirror. We show how the expectation value for the height of the particle evolves with time (thick red line). In the classical limit, which is good for a particle with a much higher jumping height, the expectation value for the height as a function of time approaches the well-known parabola. We also show the temporal change of the square of the wave function, which is the probability of finding the neutron in a certain height. One recognizes an interference pattern which is absent in a classical description. At any given time, there are heights for which the probability to find the neutron is small, leading to dark regions on a position sensitive detector. The interference pattern is moving rapidly, and its observation requires some thought. The general aim of the work with the GRANIT spectrometer is to understand the quantum regime of free fall.

To make more precise statements, we will discuss now the quantum mechanical problem of a neutron on top of horizontal mirror in a gravitational field: The Hamiltonian is given by

$$H_{3D} = -\frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial y^2} - \frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial z^2} + V(z)$$

with (see Fig. 3)

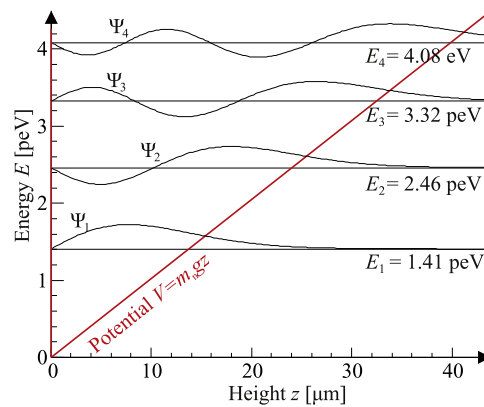
$$V(z) = \begin{cases} m_n g z, & z \geq 0 \\ \infty, & z < 0 \end{cases}$$

where  $m_n$  is the neutron mass, and  $g$  is the gravitational acceleration.

The solutions of the 3D Schrödinger equation  $H_{3D}\Phi(x, y, z) = E_{3D}\Phi(x, y, z)$  can be written in a factorized form:  $\Phi(x, y, z) = A(x)B(y)\Psi(z)$ . The different factors,  $A(x)$ ,  $B(y)$ , and  $\Psi(z)$ , fulfill one-dimensional Schrödinger equations

$$\begin{aligned} -\frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial x^2} A(x) &= E_x A(x) \\ -\frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial y^2} B(y) &= E_y B(y) \\ -\frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial z^2} \Psi(z) + V(z)\Psi(z) &= E_z \Psi(z) \end{aligned}$$

The energy in our quantum system is given by  $E_{3D} = E_x + E_y + E_z$ . The general solutions for the  $x$  and  $y$  components are superpositions of plane waves, and are not relevant for most of what is presented at the following papers. We will focus on the one-dimensional Schrödinger equation for the  $z$  component, and its solution for an energy  $E_z$  with the understanding



**Fig. 3.** The potential, as seen by UCN above a horizontal mirror, is shown as a thick red line. The lowest gravitationally bound quantum states have energies  $E_z$  of several peV, denoted  $E_1$  to  $E_4$  for the 4 lowest quantum states. Also shown are the wave functions of these quantum states as the thin black lines, denoted  $\Psi_1$  to  $\Psi_4$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Fig. 3.** Le potentiel vu par les neutrons UCN au-dessus d'un miroir horizontal est indiqué par la épaisse ligne rouge. Les états quantiques les plus bas dans le potentiel gravitationnel ont des énergies  $E_z$  de quelques peV, notées  $E_1$  à  $E_4$  pour les 4 premiers niveaux. On a représenté aussi les fonctions d'onde de ces états quantiques comme le fines lignes noires  $\Psi_1$  à  $\Psi_4$ .

that  $E_z$  is only part of the total energy  $E_{3D}$ . The lowest energy solutions of the one-dimensional Schrödinger equation for the  $z$  component are derived in quantum mechanics textbooks [1,2], and are shown in Fig. 3.

Energies of the lowest quantum states are very small, several peV, which can be compared with the upper energy limit for UCN which is in the order of 100 neV. The height of the lowest quantum states is in the order of tens of micrometers. In previous experiments in the last decade, the aim was to discover these quantum states. For the discovery, a method was used that is essentially a measurement of the height of wave functions [3–5]. The accuracy of these height measurements is not very high, about 10%. The GRANIT collaboration plans to improve on that by doing spectroscopy of the gravitationally bound quantum states; very recently, the first detection of analogous resonance transitions was reported [6]. In GRANIT, energy differences between gravitational quantum states will be determined by introducing transitions between them and by detecting the resonance frequencies, as well as by measuring interference between quantum states. This method allows a higher precision in the characterization of the gravitationally bound quantum states. Reviews of these results could be found in Refs. [7,8].

Over 50 participants from 12 countries in 4 continents attended the 2010 workshop. The first day was devoted to a detailed presentation of the new GRANIT spectrometer; the talks covered the engineering and methodical aspects as well as the theoretical analysis of its operation and first measurements to carry out using GRANIT as soon as first UCN will be delivered to the spectrometer. During the second day, we discussed various new methodical developments to be used for GRANIT in further stages, in particular real-time position-sensitive detectors of high spatial resolution, nanoparticle neutron reflectors, the new concept of a “virtual cold neutron source” for producing extremely high UCN densities, an option to use the GRANIT installation as a UCN reflectometer. Presentations during third day focused on fundamental applications of GRANIT, in particular on theoretical motivations and experiments constraining short-range forces. Competing approaches using other experiments and techniques were covered, in particular the method of neutron scattering, neutron experiments based on neutron EDM setups; studies of Casimir forces and gravity at short distances, measurements with polarized and unpolarized atoms. A broad range of alternative quantum phenomena and experiments in gravitational field were analyzed on the fourth day; an attention was devoted to common problems, experimental techniques and applications of these experiments. The recently observed phenomenon of the centrifugal quantum states of neutrons (neutron whispering gallery) as well as a photon bouncing ball was presented during the fifth day of the workshop.

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