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Novel polarized neutron devices: supermirror and spin component amplifier

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It is pointed out that evaporated multilayer structures can be used not only for neutron monochromatization, as described in the pioneering work of Schoenborn *et al.*, but also as (magnetized) mirrors with a reflectivity near to unity and a cut-off angle as high as 4–6 times the critical angle of the usual neutron mirrors. We also suggest that, using neutron spin analysers in a way that preserves wave coherence, it is possible to amplify particular components of the neutron beam polarization so as to facilitate their detection.

It has been suggested [1, 2] that multilayer evaporated 'synthetic crystals' could be used as neutron polarizers. As shown in the insert in *Figure* 1, such a structure consists of alternating layers of a ferromagnetic (M) and a nonmagnetic (V) material; the neutron scattering density of the magnetized M layer for one neutron spin direction (say 'down') equals that of the V layer. So for the down spin neutrons the multilayer has a uniform refractive index, while for the up spin neutrons it has a periodic structure giving rise to Bragg reflection and characterized by a contrast $2P$, where P is the magnetic scattering-length density of M.

It is readily found that, if 'up' spin neutron wave propagation in the layers M and V is described by the glancing angles θ_M and θ_V respectively (*Figure* 1), Snell's law reduces for the small angles in question to the form

$$\theta_M = \sqrt{(\theta_V^2 - \theta_c^2)} \tag{1}$$

where $\theta_c = \lambda\sqrt{(2P/\pi)}$ is the critical angle for total reflection of neutrons of wavelength λ at the M-V interface [3]. This shows that for glancing angles bigger than $2\theta_c$ we can take $\theta_V \simeq \theta_M \simeq \theta$, i.e. we can neglect refraction effects and apply simple Bragg theory [4] to calculate the reflectivity for such a multilayer structure with N bilayers having $d = 2d_M = 2d_V$ lattice spacing (*Figure* 1).

The in-plane dimensions of the 'elementary unit cell', being irrelevant, can be taken as unity, and straightforward evaluation of the crystallographic structure factor for such a cell gives for the first order reflection

$$F_1 = 2Pd/\pi$$

Neglecting extinction, the reflectivity of the N-bilayer structure can be taken as the square of N times the wave amplitude reflectivity of a single bilayer plane. This gives [**4**]

$$R=\frac{16}{\pi^2}N^2d^4P^2=\tfrac{1}{4}N^2(d/d_c)^4$$

where $d_c=\sqrt{(\pi/8P)}$ is the lattice spacing which would correspond to θ_c in Bragg's law; e.g. for iron $d_c\simeq 280$ Å. Hence for a high reflectivity of, say, $R=1$, which, with extinction taken into account, would give a real reflectivity of 60–70%, we need

$$N=2(d_c/d)^2 \tag{2}$$

bilayers. Note, that for $d=\frac{1}{5}\,d_c$, i.e. $\theta_{\text{Bragg}}\simeq 5\theta_\infty$, $N=50$ only. Experimental tests [**5, 6**] have confirmed the predicted high reflectivity (60–90%) and polarization (>95%).

We suggest an arrangement which extends considerably the range of applicability of the multilayer structure. A multilayer with a gradually changing lattice spacing $d(n)$ $(n=1, 2, 3\ldots)$ will give a reasonably high reflectivity for all angles up to an angle $\theta_{\max}$ equal to a few times θ_c, if for any angle within that range a sufficient number of successive layers have thickness close to the required value. Such a multilayer structure (let us call it a supermirror) would thus behave very much like a totally reflecting mirror, but with a higher cut-off angle and not quite as high a reflectivity (see *Figure* 1). The reflectivity function $R(\theta)$ of a supermirror will certainly show some fringe structure, which could be calculated by some rather lengthy numerical computation. The basic features, however, can be explored very simply (see later) and for polarizing or analysing applications the finer details of $R(\theta)$ are usually of little importance.

Similar structures have been successfully tested as wide-band, all dielectric high reflectance mirrors in ordinary optics [**7**].

The effective number of bilayers $N(n)$ contributing to the reflection at a given θ, i.e. a given $d(n)$, can be estimated by the number of those bilayers around the nth bilayer which reflect with a phase correct within $\pm 45°$ relative to that of the nth bilayer:

$$N(n)\approx -\frac{d(n)}{4}\left(\frac{\delta d(n)}{\delta n}\right)^{-1}$$

if $d(n)$ is a smooth, monotonically decreasing function of n. By virtue of *Equation* 2 we get the differential equation

$$2\left(\frac{d_c}{d(n)}\right)^2=-\frac{d(n)}{4}\left(\frac{\delta d(n)}{\delta n}\right)^{-1}$$

which has the solution

$$d(n)=2d_c/\sqrt{n}$$

Our derivation applies to $\theta>2\theta_c$, i.e. $d<d_c/2$. In the range $d_c/2<d<d_c$ refraction effects become important. They can, however, be corrected for by

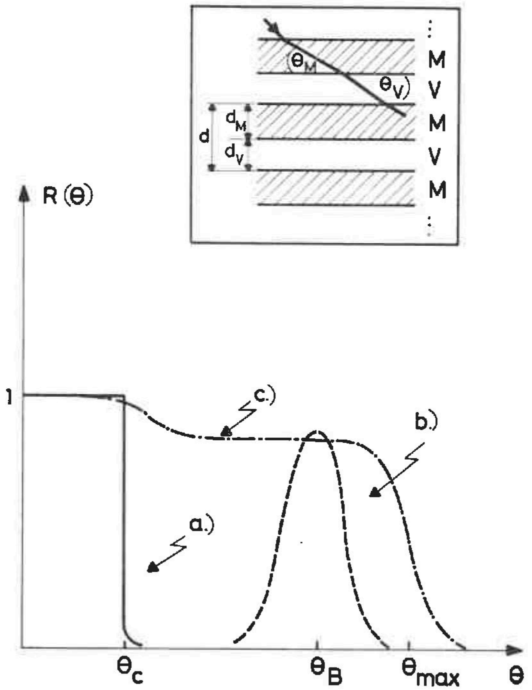

Figure 1 Schematic curves of reflectivity versus incident angle. The insert shows a part of the multilayer structure.
(*a*) Total reflecting mirrors
(*b*) Multilayer monochromators [1]
(*c*) Supermirrors

increasing the thickness of the M layers corresponding to *Equation* 1, which readily gives

$$d_M(n) = \left\{ \sqrt{\left[\left(\frac{1}{d_v(n)} \right)^2 - \left(\frac{2}{d_c} \right)^2 \right]} \right\}^{-1} = d_c/\sqrt{(n-4)}$$

while

$$d_V(n) = d_c/\sqrt{n}$$

The first M layer with $n = 5$ will be thick enough to ensure nearly total reflection for $0 < \theta < \theta_c$ also. In order to achieve good polarization in this range the refractive index of the V layer must be about equal to that of the vacuum, so as to avoid parasitic total reflection by the vacuum/V interface. A good choice of materials is vanadium for V and $Co_{0.6}Fe_{0.4}$ for M. Note that for $\theta_{max} = 5\theta_c$ only about 100 layers are needed.

The significance of these supermirror polarizers is that they promise a possibility of polarizing neutrons with no unnecessary loss in intensity. The increase of the cut-off angle to nearly 1° at as small a wavelength as 2 Å, compared to 10′ for ordinary mirrors, seems to be the quantum leap which makes possible the handling of a reasonable beam divergence, the construction of Soller type analyser systems for large-solid-angle acceptance, and the use of powerful one- and two-dimensional geometrical focussing. Non-polarizing supermirrors might also find several interesting applications, e.g. focussing small angle scattering.

The other device suggested here is based on the generalized description of the action of neutron polarizers and analysers. Since a measurement in the quantum mechanical sense happens only if a neutron is captured in the detector, the classical way of describing a polarized beam by splitting it into up and down polarized, *incoherent*, components, is generally inadequate. We should rather keep track of the coherence, i.e. the neutron-spin wave function should be written as

$$|\theta, \phi> = \cos\frac{\theta}{2}\,|\uparrow> + e^{i\phi}\sin\frac{\theta}{2}\,|\downarrow>$$

where θ and ϕ are the polar angles of the spin vector. It is this coherence that gives rise to the Larmor precessions (described by the time dependence of ϕ), which is fundamental to the neutron spin echo concept [**8**]. By the same token, the action of a neutron analyser is more correctly described by an S-matrix than by up and down reflectivities. For an analyser with negligible spin-flip cross section, the S-matrix can be written as

$$\hat{S} = A\begin{pmatrix} a_{\uparrow\uparrow} & 0 \\ 0 & e^{i\phi}a_{\downarrow\downarrow} \end{pmatrix}$$

In general ϕ and the phase of the complex coefficient A could be different for different volumes of the analyser, and thus the coherence would be lost by averaging, as it inevitably is for crystal analysers. However, evaporated mirrors and multilayers, which need only 100–200 Oe magnetizing fields, should show sufficient phase uniformity. Thus the spin state of the neutron reflected by the analyser would be given by

$$\hat{S}\Big|\theta,\phi> = A\sqrt{\left(a^2_{\uparrow\uparrow}\cos^2\frac{\theta}{2} + a^2_{\downarrow\downarrow}\sin^2\frac{\theta}{2}\right)}\Big|\theta^*\ \phi^*>$$

where θ^* and ϕ^* could be readily calculated.

An interesting application of this approach is the following case of neutron-spin-component amplification. Assume that $\theta = 180° - \epsilon$, so that there is a small deviation of the beam polarization from the quantization axis (the magnetic field direction).

Such very small deviations are of great interest, for example in the diffraction by non-centrosymmetrical magnetic structures and, allegedly, in parity-violating nuclear interaction effects. In such a case we easily find, that

$$180° - \theta^* = \epsilon^* = \sqrt{(r)}\epsilon$$

where $r = (a_{\uparrow\uparrow}/a_{\downarrow\downarrow})^2$ is the flipping ratio of the analyser. The increased deviation ϵ^* gives rise to an increased precessing spin component, which can be measured directly by applying subsequently a 90° spin turn before a second analyser used in the usual way [**8**, **9**]. This arrangement resembles the crossed polarizers in ordinary optics. The advantages of this method are illustrated in the following comparison of the number of detected neutrons N_0 needed to indicate an $\epsilon = 10^{-5}$ rad deviation for $r = 100$:

(*a*) *Classical method*: analyser only $$N_0 \simeq \frac{100}{r^2\epsilon^4} \simeq 10^{18}$$

(*b*) 90° *detection*: 90° turn + analyser $$N_0 \simeq \frac{5}{\epsilon^2} \simeq 5 \times 10^{10}$$

(*c*) *Spin component amplification*: analyser + 90° turn + analyser $$N_0 \simeq \frac{5}{r\epsilon^2} \simeq 5 \times 10^{8}$$

We note that from the point of view of the incoming beam intensity the last two methods are equivalent and 10^7 times superior to the classical one. The spin component amplifier method, however, shows the decisive advantage, namely that it requires less precision in neutron counting and the count rate is lower, for high precision neutron counting above 10^5–10^6 n/sec is a complicated, expensive, and by no means easily available procedure.

Experiments are underway to test these ideas and develop practical fabrication procedures for the large-scale use of supermirrors. The problems encountered are more severe than those in optical multilayer mirror fabrication, mainly because, for neutrons, a much higher number of much thinner layers have to be produced with precision.

When experimental results on multilayer systems with sufficiently precisely known structure become available, it will be appropriate to make more rigorous computer calculations of their behaviour. It seems, however, that the above approximate theory offers a good enough understanding of the phenomena at the present stage of experimental work.

References

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3 Bacon G E *Neutron Diffraction* (Oxford, 1962) p 118.
4 Bacon G E *Ibid* p 55.
5 Schoenborn B P. Private communication.
6 Lutter A and Mezei F. To be published.
7 See, for instance Macleod, H A *Thin Optical Film Filters* (Hilger, London, 1969) Ch 5.
There is, however, a basic difference between ordinary optics and neutron optics. Because of high refractive indices (1·3–5), all practical situations in light optics will be analogous to that of $\theta < 2\theta_c$ above, while our main interest for neutrons is the $\theta > 2\theta_c$ range. To bridge the gap between θ_c and $2\theta_c$, the sophisticated computation techniques developed in thin-film optics will be helpful, but of minor interest since they concern only the optimal choice of about 10 bilayers out of the 100 or so that we require.
8 Mezei F *Z Phys* **255** 146 (1972).
9 Mezei F *Proc of Magnetism and Magn Mat Conf* Boston 1973. (American Institute of Physics, New York, 1974) p 406.